The published version of the paper "Morbidelli R., Saltalippi C., Flammini A., Govindaraju R.S. (2018). Role of slope on infiltration: a review. Journal of Hydrology, 557, 878-886." is available at: https://doi.org/10.1016/j.jhydrol.2018.01.019

Role of slope on infiltration: a review

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Abstract

Partitioning of rainfall at the soil-atmosphere interface is important for both surface and subsurface hydrology, and influences many events of major hydrologic interest such as runoff generation, aquifer recharge, and transport of pollutants in surface waters as well as the vadose zone. This partitioning is achieved through the process of infiltration that has been widely investigated at the local scale, and more recently also at the field scale, by models that were designed for horizontal surfaces. However, infiltration, overland flows, and deep flows in most real situations are generated by rainfall over sloping surfaces that bring in additional effects. Therefore, existing models for local infiltration into homogeneous and layered soils and those as for field-scale infiltration, have to be adapted to account for the effects of surface slope. Various studies have investigated the role of surface slope on infiltration based on a theoretical formulations for the dynamics of infiltration, extensions of the Green-Ampt approach, and from laboratory and field experiments. However, conflicting results have been reported in the scientific literature on the role of surface slope on infiltration. We summarize the salient points from previous studies and provide plausible reasons for discrepancies in conclusions of previous authors, thus leading to a critical assessment of the current state of

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our understanding on this subject. We offer suggestions for future efforts to advance our knowledge of infiltration over sloping surfaces.

KEYWORDS Hillslope hydrology, overland flow, infiltration process, infiltration modeling

1. Introduction

The process of infiltration is controlled by many factors, including soil depth and geomorphology, soil hydraulic properties, and rainfall or climatic properties. The spatio-temporal evolution of infiltration rates under natural conditions cannot be currently deduced by direct measurements alone at any scale of interest in applied hydrology, therefore the use of infiltration models that rely on measurable quantities is of fundamental importance.

Even though the representation of the natural processes of areal infiltration over both flat and sloping surfaces is needed in hydrologic models, research activity has been limited to the development of local, or point, infiltration models for many years. A variety of local infiltration models for vertically homogeneous soils with constant initial soil water content and over horizontal surfaces have been proposed (Green and Ampt, 1911; Kostiakov, 1932; Horton, 1940; Holtan, 1961; Swartzendruber, 1987; Philip, 1957a,b,c; Soil Conservation Service, 1972; Smith and Parlange, 1978; Broadbridge and White, 1988; Dagan and Bresler, 1983; Corradini et al., 1994). Furthermore, for isolated storms and when ponding is not achieved instantly, extended forms of the Philip model (Chow et al., 1988), Green-Ampt model (Mein and Larson, 1973; Chu, 1978) and the Smith and Parlange model (Parlange et al., 1982) are widely used. However, for complex rainfall patterns involving rainfall hiatus periods or a rainfall rate after time to ponding less than soil infiltration capacity, these models are not directly applicable because the assumption of uniform initial soil moisture cannot be met for successive storms. Alternatively, an approach for the application of the aforementioned classical models was developed (Mls, 1980; Péschke and Kutílek, 1982; and Verma, 1982) starting from the time compression approximation proposed by Reeves and Miller (1975) for post-hiatus rainfall producing immediate ponding. However, Smith et al. (1993) by comparison with results of the Richards equation showed that the last approach was not sufficiently accurate because it neglects the soil water redistribution process which is particularly important when long periods with a light rainfall or a rainfall hiatus occur. A more general model that combines infiltration and redistribution was provided by Corradini et al. (1997) starting from the Darcy and continuity equations then combined with a conceptual representation of the wetting soil moisture profile.

Natural soils are rarely vertically homogeneous. In hydrological simulations, the estimate of effective rainfall can be reasonably schematized by a two-layered vertical profile (Mualem et al., 1993; Taha et al., 1997). A general semi-analytical/conceptual model for crusted soils was formulated by Smith et al. (1999), and was extended by Corradini et al. (2000) to represent infiltration and reinfiltration after a redistribution period under any rainfall pattern and for any two-layered soil where either layer may be more or less permeable than the other. For a much more permeable upper layer, and under more restrictive rainfall patterns, a simpler semi-empirical/conceptual model was presented by Corradini et al. (2011). Under conditions of surface saturation, a simple Green-Ampt based model was proposed (Chow et al., 1988).

In applied hydrology, upscaling of point infiltration modeling to the field scale is required to estimate the areal-average infiltration. This is a complex task because of the natural spatial heterogeneity of hydraulic soil properties (Nielsen et al., 1973; Warrick and Nielsen, 1980; Greminger et al., 1985; Sharma et al., 1987; Loague and Gander, 1990) and particularly of the soil saturated hydraulic conductivity (Russo and Bresler, 1981; 1982) that may be assumed as a random field with a lognormal univariate probability distribution. Some models representing infiltration at the field scale have been proposed with saturated hydraulic conductivity, K_s, assumed as a random variable at the soil surface for both uniform (Smith and Goodrich, 2000; Govindaraju et al., 2001) and non-uniform soils (Corradini et al., 2011; Govindaraju et al., 2012) in the vertical direction. Further, models were developed to describe the effects of a joint horizontal variability of K_s and rainfall rate r (Wood et al., 1986; Castelli, 1996; Govindaraju et al., 2006; Morbidelli et al., 2006), and of the spatial variability of initial soil moisture content θ_i (Smith and Goodrich, 2000). The role of the heterogeneity of θ_i, combined with uniform values of K_s and r or with K_s randomly variable, has been widely analyzed for different spatial scales (Brontsert and Bardossy, 1999; Morbidelli et al., 2012; Hu et al., 2015).

Most of the above-mentioned models consider a horizontal soil surface or one with a low slope that does not affect the infiltration process. However, in most real situations infiltration occurs in surfaces characterized by different gradients (Beven, 2002; Fiori et al., 2007) and the role of surface slope on infiltration is not clear. In fact, the results obtained by some theoretical (Philip, 1991; Chen and Young, 2006; Wang et al., 2018) and experimental investigations (Nassif and Wilson, 1975; Sharma et al., 1983; Poesen, 1984; Cerdà and García-Fayos, 1997; Fox et al., 1997; Chaplot and Le Bissonnais, 2000; Janeau et al., 2003; Assouline and Ben-Hur, 2006; Essig et al., 2009; Ribolzi et al., 2011; Patin et al., 2012; Lv et al., 2013; Morbidelli et al., 2015; Mu et al., 2015; Khan et al., 2016; Morbidelli et al., 2016) lead to rather contrasting conclusions, suggesting an improved understanding and modeling of infiltration over sloping surfaces is required.

The overall intent of this paper is to highlight the state of the art on the slope-infiltration relationship and provide guidance for future developments on the basis of available results.

2. Theoretical Formulations

2.1 Analytical formulation

Let consider a long planar hillslope consisting of a homogeneous isotropic soil, with slope angle γ, and Cartesian rectangular spatial coordinates x and z, with x and z positive in the horizontal downslope direction and in the downward vertical direction, respectively. Let introduce also the rotated coordinates (x*, z*), as defined in Fig. 1, for explicitly accounting for slope:

$$x^* = x\cos(\gamma) + z\sin(\gamma) \tag{1}$$

$$z^* = -x\sin(\gamma) + z\cos(\gamma) \tag{2}$$

insert here Fig. 1

According to Philip (1957b; 1969) the equation that governs unsaturated soil water movement may be expressed in the form:

$$\frac{\partial\theta}{\partial t} = \nabla\cdot\left(D\nabla\theta\right) - \frac{dK}{d\theta}\frac{\partial\theta}{\partial z} \tag{3}$$

with t the time, θ the volumetric soil water content, D the soil diffusivity, and K the hydraulic conductivity, being D and K typically nonlinear functions of θ.

We first examine the dynamics of infiltration and downslope water transport by searching for the solution of eq. (3) under the conditions:

$$t=0 \qquad z^*>0 \quad \theta=\theta_0 \tag{4}$$

$$t>0 \qquad z^*=0 \quad \theta=\theta_1 \tag{5}$$

where θ_0 is the uniform initial volumetric soil water content, and θ_1 is the value of θ associated to the capillary head ψ_1 at which water is available at the soil surface $z^*=0$. Under the hypothesis of negligible depth of free water excess on the hillslope, $\psi_1=0$ and θ_1 is the saturated volumetric soil water content.

Expressing eq. (3) in terms of x* and z*, we derive:

$$\frac{\partial\theta}{\partial t}=\nabla\cdot\left(D\nabla\theta\right)-\frac{dK}{d\theta}\left[\frac{\partial\theta}{\partial x^*}\sin\left(\gamma\right)+\frac{\partial\theta}{\partial z^*}\cos\left(\gamma\right)\right] \tag{6}$$

Except for a small upper area of the hillslope, the relevant solution of eq. (6) subject to the conditions of eqs. (4) and (5) is basically independent of x* and depends only on z* and t. On this basis, we may rewrite eq. (6) as:

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z^*}\left[D\frac{\partial\theta}{\partial z^*}\right]-\frac{dK}{d\theta}\frac{\partial\theta}{\partial z^*}\cos\left(\gamma\right) \tag{7}$$

We note that eq. (7) subject to eqs. (4) and (5) is formally identical to the classical one-dimensional infiltration equation, if K is substituted by $K\cos(\gamma)$.

Reverting to the non-rotated axes x and z, eq. (7) becomes:

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left[D\frac{\partial\theta}{\partial z}\right]\sec^2(\gamma)-\frac{dK}{d\theta}\frac{\partial\theta}{\partial z} \tag{8}$$

The vertical component of the unsaturated water flux, v, may be expressed as:

$$v=K-D\frac{\partial\theta}{\partial z} \tag{9}$$

while its value at the surface z=x sin(γ), v_0, may be considered the infiltration rate of standard hydrologic practice, provided by:

$$v_0=K_1-\left(D\frac{\partial\theta}{\partial z}\right)_{z=x\sin(\gamma)} \tag{10}$$

where K_1 stands for $K(\theta_1)$.

We can introduce analogous quantities in terms of rotated coordinates (x*, z*), denoting with v_n the infiltration rate normal to the hillslope:

$$v_n=K\cos(\gamma)-D\frac{\partial\theta}{\partial z^*} \tag{11}$$

and its value for z*=0, v_{n0}:

$$v_{n0}=K_1\cos(\gamma)-\left[D\frac{\partial\theta}{\partial z^*}\right]_{z^*=0} \tag{12}$$

The theoretical approach above descripted states that the gravitational effect in the direction normal to the slope decreases by a factor cos(γ) and entails a small reduction of the infiltration rate, while capillary forces are invariant.

Along these lines, Philip (1991) suggested an analytical series solution and two simplified relationships for different time intervals. In order to explicit the comparison with infiltration rate for a horizontal soil surface he proposed these ratios:

$$\lim_{t\to 0}\frac{v_{n0}(\gamma)}{v_{n0}(0)}=1 \tag{13}$$

$$\lim_{t\to \infty}\frac{v_{n0}(\gamma)}{v_{n0}(0)}=\cos(\gamma) \tag{14}$$

At short times, when the capillary forces drive the process, the infiltration rate normal to the slope doesn't depend on the slope angle (see eq. 13). On the contrary, at very long times, when only gravitational forces play a role, v_{n0} reduces with cos(γ), being the gravity force only vertical.

2.2 Conceptual formulation

From eq. (7), it may be derived that except for the difference of coordinate system, the only change needed for describing infiltration over a sloping surface is to substitute K with K cos(γ), and only the component of the water flux normal to the soil surface is changed by cos(γ). On the contrary, the downslope component of gravity does produce flow, but it does not modify the soil moisture profile along the normal direction on a planar slope because the flow field is independent of x*. As a consequence, as an approximation, the Green-Ampt (GA) model can be rearranged including the same modification.

On the basis of Darcy's law, Chen and Young (2006) proposed a modified version of the GA model for a sloping surface under ponded conditions as:

$$v_n = K_e \frac{z_f^* \cos(\gamma) + s_f + H}{z_f^*} \tag{15}$$

with K_e the effective saturated hydraulic conductivity, z_f* the depth of the wetting front along the direction normal to the slope surface, s_f the wetting front capillary head, H the ponding water head on the surface, and z_f*cos(γ) the gravitational head at the wetting front (see Fig. 2). Physically, each variable is considered to remain invariant along the downslope direction.

insert here Fig. 2

Being the ponding depth on a sloping surface usually small if compared to the wetting front capillary head s_f, in their theoretical analysis Chen and Young (2006) treated it as a revision to s_f. The cumulative infiltration depth in the normal direction, I_n, can be determined as:

$$I_n = (\theta_s - \theta_i) z_f^* \tag{16}$$

Taking the derivative of I with respect to time and substituting into eq. (15) yields:

$$\frac{dz_f^*}{dt} = \frac{K_e}{(\theta_s - \theta_i)} \frac{z_f^* \cos(\gamma) + s_f + H}{z_f^*} \tag{17}$$

By integration with respect to time eq. (17) provides:

$$t=\frac{\left(\theta_s-\theta_i\right)}{K_e\cos(\gamma)}\left[z_f^*-\frac{\left(s_f+H\right)}{\cos(\gamma)}\ln\frac{z_f^*\cos(\gamma)+s_f+H}{s_f+H}\right] \tag{18}$$

Substituting eq. (16) into eq. (18) yields the following simplified form:

$$K_e t\cos(\gamma)=I_n-\frac{SM}{\cos(\gamma)}\ln\left[1+\frac{I_n\cos(\gamma)}{SM}\right] \tag{19}$$

with $S=s_f+H$ and $M=\theta_s-\theta_i$.

Equation (19), that is the key equation of the GA model, describes in an implicit way the variation in time of cumulative infiltration depth. Chen and Young (2006) expanded the second term of the right-hand side of eq. (19) with a Taylor series on I_n around point $I_n=0$ and keeping the first two terms in the series yielded:

$$K_e t\approx\frac{1}{2}\frac{I_h^2\cos^2(\gamma)}{SM} \tag{20}$$

with $I_h=I_n/\cos(\gamma)$ the cumulative depth in the vertical direction. This solution is valid only for small time according to the converging range of Taylor series. The variable I_h compares infiltration on a slope to that on a horizontal surface, but with the same horizontal projection lengths, as:

$$\frac{I_h(\gamma)}{I_h(0)} = \frac{1}{\cos(\gamma)} \tag{21}$$

This implies that the sloping surface increases the infiltration at small t by a factor 1/cos(γ). For t→∞, or for high infiltration depths, eq. (19) can be approximated as:

$$K_e t \approx I_h \tag{22}$$

which highlights that the slope effect reduces with time and vanishes at very long t.

Therefore, according to Chen and Young (2006), infiltration at small times is controlled by capillary forces, which would be independent of slope angle in case of homogeneous and isotropic soils. However, when the slope angle increases the slope length increases, and consequently also the total infiltration volume increases. For long t (or large I_n), the gravity becomes the control mechanism, and the normal flux would be reduced by the factor cos(γ). This effect cancels with increasing slope length, and the net slope effect essentially vanishes.

The results by Chen and Young (2006), obtained through a modified form of the GA model, depend by the condition of identical slope horizontal projection lengths. In fact, despite apparently in contrast to the above results by Philip (1991), the modified GA model was compared to a solution of Richards' equation on a sloping surface and was shown to match well.

Wang et al. (2018) proposed a new theoretical formulation involving the estimate of ponding time and infiltration on hillslopes under both steady and unsteady rainfall conditions. The infiltrability equation was developed by integration of Darcy's law for sloping surfaces and incorporating the flux-concentration equation (Sivapalan and Milly, 1989) as:

$$t = \frac{1}{\cos^2(\gamma)} \int_{\theta_i}^{\theta_s} \frac{D(\theta)(\theta - \theta_i)}{(N - K_i)^2 F} \left[\ln \frac{i_n - N\cos(\gamma)}{i_n - K_i \cos(\gamma)} + \frac{(N - K_i)\cos(\gamma)}{i_n - N\cos(\gamma)} \right] d\theta \tag{23}$$

where i_n is the infiltrability normal to the surface slope, t is time, $F=(\theta-\theta_i)/(\theta_s-\theta_i)$ and $N=K_i+(K(\theta)-K_i)/F$. Equation (23) leads to a scaling relation between infiltration on sloping surfaces and horizontal surfaces: under ponding conditions the normal infiltration rate on a sloping surface reduces to that on horizontal surface scaled through a proportionality factor equal to $\cos(\gamma)$, by taking $1/\cos^2(\gamma)$ times the time for the horizontal plane:

$$t(\gamma) = \frac{t(0)}{\cos^2(\gamma)} \tag{24}$$

where $t(\gamma)$ and $t(0)$ are the times for slope and horizontal plane, respectively, associated to the same value of infiltrability.

For time to ponding, t_p, Wang et al. (2018) proposed the following equation:

$$t_p = \frac{1}{\cos^2(\gamma)} \int_{\theta_i}^{\theta_s} \frac{D(\theta)(\theta - \theta_i)}{(r - K_i)[F(r - K_i) - (K(\theta) - K_i)]} d\theta \tag{25}$$

where r is a constant rainfall rate, and they found that the ponding time for hillslopes can be estimated from that of horizontal plane through eq. (24).

After ponding and under steady rainfall conditions, eq. (23) could be solved for temporal evolution of infiltration. Alternatively, in order to avoid the required computational effort, Wang et al. (2018) selected an explicit empirical relation that links the infiltration rate with time including the slope effect as:

$$i_n = at^b + K_s \cos(\gamma) \tag{26}$$

where a and b are parameters related to initial soil moisture content, soil type and rainfall intensity. A procedure to extend the approach to unsteady rainfall was also included. The proposed approach was validated by comparison with results obtained by Hydrus 1D and by the modified GA model for sloping surfaces (Chen and Young, 2006).

3. Experimental Evidence

Investigations to address the understanding of the effect of surface slope on the infiltration process have been performed through some experiments in both laboratory (Nassif and Wilson, 1975; Poesen, 1984; Fox, et al., 1997; Assouline and Ben-Hur, 2006; Essig et al., 2009; Lv et al., 2013; Morbidelli et al., 2015; Mu et al., 2015; Khan et al., 2016; Morbidelli et al., 2016) and field (Sharma et al., 1983; Cerdà and García-Fayos, 1997; Chaplot and Le Bissonnais, 2000; Janeau, 2003; Ribolzi et al., 2011; Patin et al., 2012) settings. Even though these studies provided conflicting results as to whether infiltration increase or decrease with slope, all of them provided useful insights.

3.1 Laboratory simulations

Useful conclusions were derived by Essig et al. (2009) and Morbidelli et al. (2015, 2016) by examining results obtained from a long series of laboratory experiments conducted with an experimental system consisting of:

- a soil box with characteristics shown in Fig. 3 and the slope adjustable in the range 1-25°;
- study soils of thickness 67 cm obtained from natural soils, over 7 cm of gravel to speed the drainage of the percolated water from the soil;

- a rainfall simulator, based on sprinklers of water under pressure provided by a pump, that produces a uniform rainfall distributed over the soil surface of intensity calibrated in advance and chosen through the appropriate combination of sprinkler type and water pressure;
- tipping bucket sensors that provided continuous surface and deep flow at the downstream soil boundary;
- time domain reflectometry sensors that collect continuous observations of the average soil water content at different depths in two vertical profiles.

A first set of 50 experiments was discussed by Essig et al., (2009) with three different soil types (a clay loam, a loam and a sandy loam; each soil type designation hereinafter is based on the USDA classification), slopes ranging from 1° to15° and rainfall rates in the range 10-30 mmh^{-1}. For each experiment, rainfall was applied to reach soil saturation throughout the box. Surface runoff and deep flow were collected for up to 24 h. It was observed that the time to ponding increased as rainfall intensities decreased. Surface flowrate (normalized by the average rainfall rate) versus time were used to compare runoff rates for different slopes, soil types, and rainfall rates. For the clay loam soil, the slope had a great positive effect on the steady state surface flow. Based on kinematic wave theory, the time of concentration should be under one hour. However, the duration of the observed receding limb of surface flow was longer than expected after the rainfall was turned off. This tail was more prominent for steeper slopes and less evident for coarse soils. The normalized infiltration rates for the clay loam soil (Essig et al., 2009) suggested the presence of soil water outflow near saturation (i.e. a seepage face), but this was not as obvious for the loam and the sandy loam soils. On the basis of the above theoretical formulations, one would expect that the normalized steady deep flow should only vary by a factor of cos(γ), but the results reported in Essig et al. (2009) did not conform to this expectation and the steady deep flow decreased by much larger amounts with slope angle. Three different mathematical models were employed to explain the measured data. The authors proposed an effective saturated hydraulic conductivity, that empirically accounts for slope effects, to obtain realistic agreements with measurements of overland flow, deep flow and water content at different depths.

insert here Fig. 3

After an on overall analysis of the observed results, Essig et al. (2009) postulated that relationships between rainfall and surface runoff (and deep flow) were being influenced by the following aspects:

- The walls of the sand box enforced a condition of zero flux normal to the boundary. This might be altering the flow pattern sufficiently so that the flow was strongly influenced by wall effects.
- A longer recession tail, particularly for the steeper slopes in the clay loam soil suggested the existence of return flow from saturated areas. This return flow, if it exists, would be more noticeable for steep slopes and for fine-textured soils.
- The infiltration of water moving on the sloping soil surface was different from the case of infiltration over a flat surface where the ponded water had no momentum in the direction tangential to the slope.

To shed more light on these issues, Morbidelli et al., (2015) carried out a second set of 15 laboratory experiments with two important modifications:

- surface runoff assimilation by a technique which enabled to perform measurements in the middle of the box (along the slope);

- much lower values of the ratio r/K_s changed from ~2-3 to ~0.7-1.3.

The first modification allowed the authors to eliminate the effect of the downstream boundary on the separation between surface and subsurface flows. The second modification extended the investigation to commonly observed rainfall rates and assured the absence of both erosion and sealing layer.

Results by Morbidelli et al. (2015) indicate that, even for moderate rainfall rates, the variation of the subsurface flow with γ is very evident. Their results, which refer substantially to conditions with prevailing gravitational effects, do not agree with any theoretical result, and further support the trends shown by Essig et al. (2009). Morbidelli et al. (2015) suggested the existence of a relation between the decrease of the steady deep flow with γ and the shear stress at the soil surface as a basis for estimating an effective saturated hydraulic conductivity to be used in the existing infiltration models for horizontal surfaces. In this context, the role of the surface roughness remained an open problem to be addressed through specific field and laboratory experiments. Furthermore, for bare sloping surfaces Morbidelli et al. (2015) noted the production of surface runoff even for $r<K_s$

In a subsequent set of laboratory experiments, Morbidelli et al. (2016) provided experimental evidence on the role of roughness in the relation between infiltration and slope angle. Twenty-eight new simulations were performed by the same experimental system but using a grassy soil surface. Morbidelli et al. (2016) provided fresh evidence of the relationship between γ and infiltration rate on a grassy soil and remarked the significant differences existing between bare and grassy soils. Their laboratory simulations highlighted that the effect of slope gradient on infiltration rate was greatly reduced by the grassy soil surface with a smaller decrease of the steady infiltration rate for γ in the range 1-15°. More specifically, their results provided evidence of the existence of an effective soil saturated hydraulic conductivity, $K_e(\gamma)$, associated with rainfall rates that yielded surface runoff and steady deep flow for each γ

value. This quantity decreased from ~K_s for γ=1° down to ~0.8 K_s for γ=10°. The divergence with results computed by theoretical formulations is much more significant in the case of a bare soil for which, from Morbidelli et al. (2015), $K_e(\gamma=10°)\approx 0.2\ K_s$. A common feature of slopes with grassy soils and bare soils concerns the production of surface runoff for $K_e<r<K_s$, that is in unsaturated soils. Furthermore: (1) the trials presented in the work for a grassy soil by Morbidelli et al. (2016) coupled with those earlier described for bare soils by Essig et al. (2009) and Morbidelli et al. (2015) suggest that the magnitude of the effects of γ on the gravitational component of infiltration rate is determined by a mechanism independent of the formation of rills or a sealing layer. In fact the experiments were not affected by the last two processes, which influenced several previous investigations; (2) the formation of a two-layered soil due to the grass growth cannot describe the effects of γ on the infiltration rate; and (3) the relation between γ and infiltration rate is strictly dependent on the surface roughness.

Along the same lines, Nassif and Wilson (1975), Fox et al. (1997), Mu et al. (2015) and Khan et al. (2016) conducted laboratory experiments and found that the infiltration rate decreased with increasing slope angle. Nassif and Wilson (1975) used a laboratory apparatus of horizontal dimensions 6.15 x 4.10 m and soil depth 0.22 m and an artificial rainfall generator to collect data about runoff and infiltration in different soil types and for different surface slopes. Among other results, they emphasized that the increase of slope had little effect on runoff in relatively impermeable soils and significant effects on natural soils, with an increase up to 16% and 24% in bare and grassed surfaces, respectively. They also observed a critical slope over which the peak runoff became invariant.

Fox et al. (1997) used a sandy loam soil, susceptible to surface crusting (Fox, 1994), packed in a 100 x 40 x 10 cm trays by applying successive layers with light compaction and smoothing between layers. The trays were set at different slope angles in the range 1.5-21.5°, subjected to rainfall rate in the range 38.2-56.3 mmh^{-1}, each with duration of 90 minutes. Infiltration was calculated from the overland flow rate, and mictotensiometers and micromorphological analysis were used to characterize seal formation. Infiltration rate decreased with increasing slope angle until 11.5° and remained unchanged at steeper slope angles. The image analysis of pore characteristics clearly suggested that the slope angle had no significant impact on surface seal development. The estimated change in mean overland flow depth with slope angle was in the order of about 1 mm, and from visual observations it never appeared to exceed a few mm within any flow thread. In the presence of a sub-seal pressure head from about -100 mm to -200 mm, an additional positive pressure head at the surface of 1 mm is insignificant and in itself would not increase the infiltration rate. So the change in overland flow depth did not add sufficient pressure head to account for the change in observed infiltration rate. However, the increase in depth would be sufficient to submerge significant portions of the high locations of the microtopography. Hence, small changes in flow depth may increase the infiltration rate by submerging areas of slightly greater hydraulic conductivity around the more stable aggregates.

Mu et al. (2015) conducted laboratory simulations to study the effects of various factors including the slope gradient on the runoff generation mechanism in a soil cultivated with spring maize during three growing stages (jointing stage, tasseling stage and mature stage). They selected a sandy loam soil packed in a 200 x 50 x 60 cm steel bin, with slope variable in the range 0°-30°. Through some experimental trials with different combinations of rainfall intensity, slope gradient, and growing phase, they found that the overland flow and the cumulative runoff increased with the increase of rainfall rate and slope in each vegetation stage. Within a single growing stage of spring maize, they found that the runoff coefficient increased with increasing slope because of to a decrease in the soil infiltration rate, and proposed an empirical relationship that provides the runoff coefficient by a logarithmic dependency on the sine of slope. Largest runoff coefficients were found for the mature stage with values increasing from 0.22 to 0.41 for slopes changing from 5° to 20° under a rainfall rate of 0.5 mm/min.

Khan et al. (2016) adopted an artificial rainfall generator with 324 nozzles and runoff trays that could be adjusted to the desired slope angle in the range 5°-25°. They conducted 72 simulation runs under numerous combinations both in mulched and un-mulched silty loam soils with rainfall intensities ranging between 33 mmh^{-1} and 120 mmh^{-1}. The duration for each rainfall event was 1 h and steady conditions were never reached. Khan et al. (2016) found that infiltration rate decreased with an increase in slope and increased with an increase in rainfall intensity. They concluded that the effect of rainfall intensity on the infiltration rate changed with the slope angle due to the creation of different micro-relief features and that in mulched soil the water infiltration rate significantly increased with an increase in rainfall intensity at all slope angles because of the uniform surface conditions under the mulch layers. These trends agreed with those showed by Essig et al. (2009) and Morbidelli et al. (2015, 2016), with the magnitudes of the reduction in infiltration with slope much larger than expected from all theoretical studies.

On the other hand, Poesen (1984) found that the infiltration rate increased with increasing slope angle. In fact, his experimental results were characterized by a decrease in runoff with increasing slope for soils subjected to surface crust formation. Specifically, the runoff coefficient was found to be higher for a 2% slope than a 15% slope, and the mean percolation coefficient was lower for the 2% than 15% slope. These results, incidentally very similar to results by Assouline and Ben-Hur (2006), indicated a positive relationship between slope and infiltration rate, which was more pronounced for soils with water content at field capacity or greater. Poesen (1984) attributed the decreased runoff to either a thinner soil crust or increased rill erosion on the steeper slopes and concluded that 1) surface sealing is inversely related to slope, so steeper slopes would have a thinner compressed soil layer than flatter slopes and would be more prone to infiltration, 2) steeper slopes erode more quickly and increased erosion forms deeper rills, thus the surface area over which infiltration can occur becomes larger, and 3) in the absence of erosion and surface sealing the slope would not be expected to affect the infiltration process.

Of particular interest are also the analyses conducted to clarify the effect of different slope angles on water movement during unsaturated stages. For example, laboratory experiments were carried out by Lv et al. (2013) considering the redistribution process in a variable-slope soil tank (from 0° to 30°), with a homogeneous and isotropic sandy loam soil. The results showed that, increasing the slope became larger the gradient of soil water potential in the lateral downslope direction parallel to the slope surface. It was concluded that the water movement in the lateral downslope direction parallel to the slope surface was more sensitive to changes in the slope angle than the component normal to the slope surface. Lv et al. (2013) also observed that the influence of the slope angle on the flow component normal to the slope surface was greatest at a certain depth into the soil .

3.2 Field experiments

With the main objective to analyze the effect of slope angle on interrill erosion for different plot sizes and to identify possible detachment and transport processes involved in the relations between slope, rain characteristics and plot sizes, Chaplot and Le Bissonnais (2000) conducted a detailed study in an experimental field located in the northwest part of the Paris basin. The site was characterized by silty loam soils very susceptible to soil crusting. The experimental field was about 100 m in length and located in the middle of a convexo-concave catena with slope gradients of about 2%, 4%, 8% and 2% from top to bottom. Three 1 x 1 m bounded plots were established at three positions along the catena with slopes 2%, 4% and

8%. Two additional 2 x 5 m bounded plots were selected at the 4% and 8% slope positions. Six natural rainfall events (total rainfall depth of about 100 mm, intensities in the range 1.31-8.00 mmh^{-1}), in addition to artificial rainfalls with intensities up to 50 mmh^{-1}, were considered. Flow depth and detention capacity were very low because of initially smooth crusted surfaces. Infiltration rates were computed from runoff rates and rainfall intensity. As slope increased from 2% to 8%, infiltration decreased from 1 to 0.5 mmh^{-1} for low intensity natural rains, from 4 to 1 mmh^{-1} for 8 mmh^{-1} natural rain, and from 15 to 5 mmh^{-1} for 50 mmh^{-1} artificial rainfall. Increase in flow velocity with slope steepness and length was considered to be a possible explanation of the slope effect on runoff. No rills occurred in this study. The slope effect on runoff was substantially the same for both the 10 m^2 and 1 m^2 plots. Scale effect on runoff does not seem to be important for the range of scales adopted.

Earlier, Sharma et al. (1983) had conducted field experiments for a period of six years on a representative loamy sand soil. Plots with slopes ranging from 0.5% to 10% and slope lengths ranging from 5.12 m to- 14.5 m were used. Under natural rainfall depths, they found that with dry antecedent soil conditions infiltration was governed by rainfall depth, whereas with wet antecedent soil conditions raindrop impact which formed a crust over the soil surface was the deciding factor. Infiltration decreased significantly with increasing slope due to reduction in the time available for rainfall to infiltrate, but a slope length had no substantial effect.

Opposite conclusions were reached by Janeau et al. (2003) about infiltrability and slope gradient under field conditions through experiments on a gravelly loamy soil occupying the upper half of a cultivated convex hill in northern Thailand. Fifteen 1 x 1 m plots with slope gradients in the range16-63% were selected, and different artificial rainfall patterns were chosen. The steady final infiltration rate increased sharply with increasing slope gradient. Microaggregates tended to behave like sand and became tightly packed on moderate slopes (packing crust). From these results Janeau et al. (2003) deduced that the vertical component of kinetic energy (greater on moderate slope) had a prevailing role. Furthermore they asserted that 1) on steep slopes the horizontal component of the kinetic energy was transformed into shear stress, impeding the development of crusts so that water could still infiltrate, and 2) on steeper slopes, the water film was thinner, thereby limiting the role of splash. Janeau et al. (2003) concluded that the relationship between slope gradient and infiltrability is affected by the soil nature and should be investigated considering surface crusting processes.

Infiltration experiments conducted by Ribolzi et al. (2011) in two small plots characterized by very different slopes with rainfall intensities in the range 60-120 mmh^{-1} produced results similar to that previously described (Janeau et al., 2003); final infiltration rates of 6 mmh^{-1} and 21 mmh^{-1} were obtained for the 30% and 75% slopes, respectively. These experiments confirm the hypothesis that higher effective rainfall intensity is responsible for the development of less permeable erosion crust under low slope gradients whereas more permeable structural crust develop under high slope gradients.

Finally, during a long-term survey of a small agricultural basin in Lao, Patin et al. (2012) achieved interesting results as part of analyses of surface runoff formation at plot scale. They observed that infiltrability decreased when slope increased up to approximately 50% and increased with slope for steeper slopes (>50%), probably due to two opposite trends: when the soil is covered, at least partly, with vegetation, crust cover remains limited and infiltration can decrease normally with increasing slope. When the soil is bare, as commonly observed for steep slope, the opposed relationship is achieved due to lesser development of crust.

4. Assessment and Future Developments

Table 1 provides a summary of the experimental work dealing with infiltration on sloping surfaces and includes efforts on theoretical (conceptual) analyses along with experiments carried out in both laboratory and field settings. This article suggests that the interaction between surface and subsurface waters seems to be more nuanced than would be suggested by our current understanding of infiltration processes. The role of slope on infiltration is complicated by many confounding factors such as rainfall (or applied water) intensity, microtopography, vegetation, soil texture, and vertical and horizontal heterogeneity in soil properties. Consequently, the conclusions from previous studies have been mixed and even contradictory, perhaps because the results of the experiments studying the role of slope on infiltration were also influenced by one or more of these confounding factors in different ways.

There is growing laboratory evidence that the reduction in infiltration occurs beyond the cos γ factor expected during steady saturated conditions. This is especially prominent for infiltration over bare slopes and for clay soils, and becomes less prominent for vegetated surfaces and sandy soils. When studies have reported an apparent increase in infiltration, this has perhaps been due to the formation of a sealing layer, or because of some complications introduced through rill formation. Experiments that were designed to eliminate these effects have reinforced this behavior of increased infiltration beyond what is expected from a cosine of the slope angle. The number of repeated (and repeatable) experiments from multiple research groups, suggests that this trend is not merely measurement error or experimental aberration, and warrants careful scrutiny.

The development and testing of new theoretical underpinnings for to describe infiltration on sloping surfaces is needed to move the science forward. While researchers have postulated that tangential velocities at the soil surface, the increased depth of water in microtopographic depressions, or shear stress exerted by the overland water on the soil surface might cause this apparent increase in infiltration rates, a comprehensive theory is still lacking. Studies have suggested the idea of an “effective saturated hydraulic conductivity” to offer an empirical correction, but have not offered a theoretical basis that would allow this understanding to be extended to other cases beyond those covered by the range of experiments. A physical explanation followed by supporting mathematical formulation would be an important step in either supporting the hypothesis of increased infiltration with slope, or provide refutation of this notion and provide an explanation for the experimental results that support this hypothesis. Such a theory would also serve to inform us as to what future experiments to conduct and what to measure to close the gaps in our understanding.

Researchers conducting field-scale experiments have to contend with the natural spatial variability in hydraulic properties of soils and the associated role of run-on. Experimental efforts often measure averaged quantities such as rainfall and runoff from the entire field, i.e. integrated responses. As was noted earlier, it is very difficult to make independent point-scale measurements at all space and time scales, and therefore need both a good local model as noted in the previous paragraph, and a description of the nature of the spatial variation of hydraulic properties to perform upscaling studies.

Extensions of infiltration models to watershed scales are, of course, further complicated with variability at more spatial scales and the role of channel networks, and current practice relies on calibration and corroboration approaches at these scales. This operational process will have to be the state of practice until a meaningful way of upscaling knowledge from sub-grid scales is developed.

insert here Tab. 1

Acknowledgment

This research was mainly financed by the Italian Ministry of Education, University and Research (PRIN 2015).

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Fig. 3 – Schematic representation of the laboratory system with variable slope angle adopted by Essig et al. (2009), and Morbidelli et al. (2015; 2016).

Figure 1

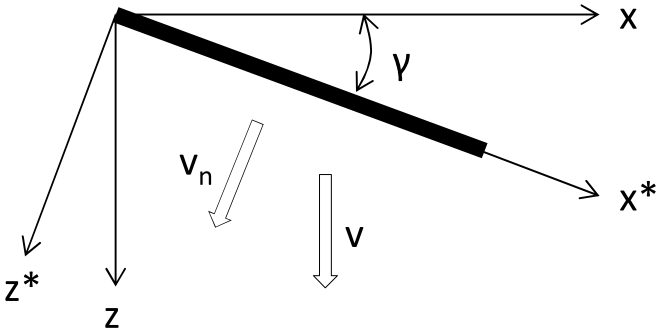

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Figure 2

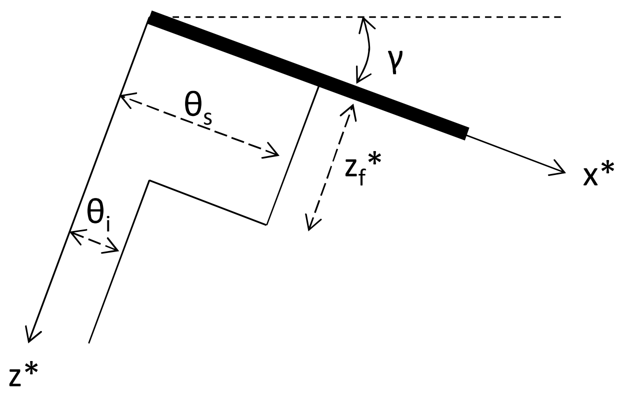

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Figure 3

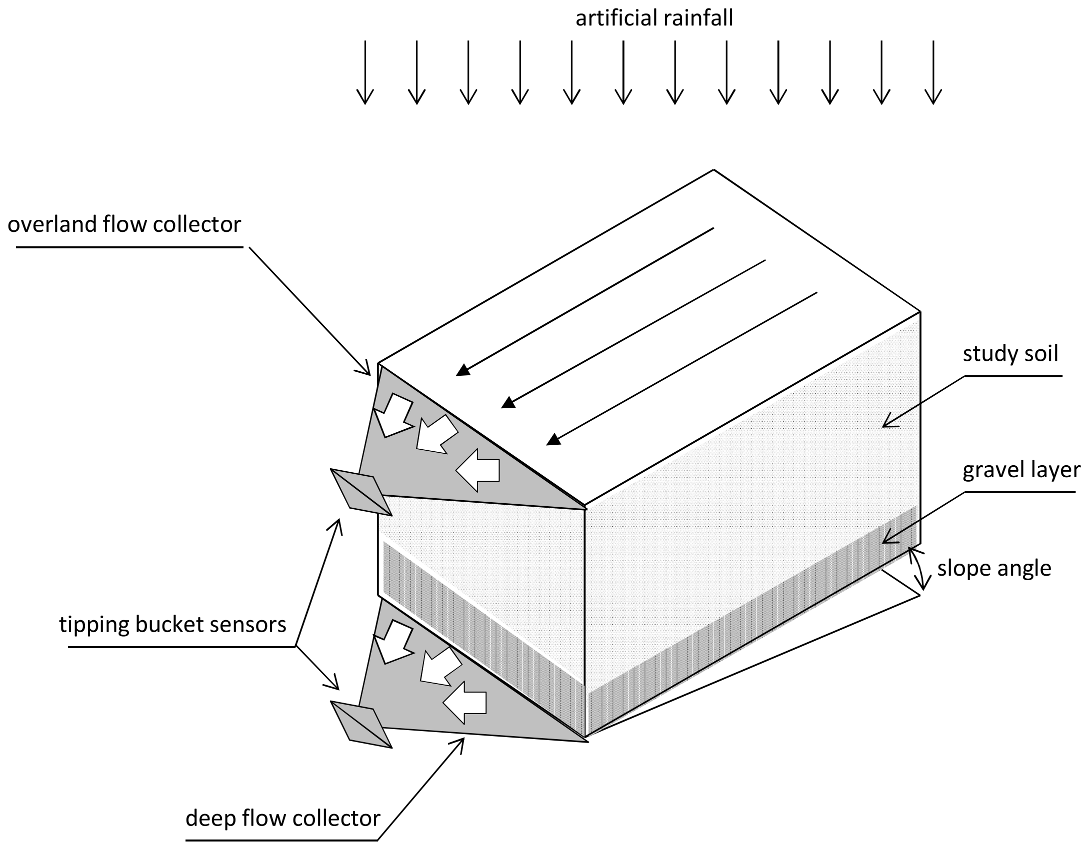

Fig. 3 – Schematic representation of the laboratory system with variable slope angle adopted by Essig et al. (2009), and Morbidelli et al. (2015; 2016).

Table 1 – Main characteristics of scientific studies analyzing the role of slope on infiltration

Authors	**Year**	**Analysis type**	**Soil type**	**Slope range**	**Main insight**
Nassif and Wilson	1975	experimental (laboratory)	sandy clay, peat, standard agricultural soil	0%-32%	vertical infiltration decrease with increasing slope (it exists a critical slope beyond which infiltration remain unchanged)
Sharma et al.	1983	experimental (field)	loamy sand	0.5% - 10%	vertical infiltration decrease with increasing slope (less time for rainfall to infiltrate)
Poesen	1984	experimental (laboratory)	sandy, silty	2° - 20°	vertical infiltration increase with increasing slope (thinner crust and more rill erosion)
Philip	1991	theoretical (analytical)	-	-	normal infiltration decrease with increasing slope (by a factor cos(γ))
Cerdà and García-Fayos	1997	experimental (field)	loam, silty-loam, silty-clay, silty clay-loam	2° - 55°	vertical infiltration independent by slope angle
Fox et al.	1997	experimental (laboratory)	sandy loam	1.5° - 21.5°	vertical infiltration decrease with increasing slope until ~11° (interaction between flow depth and submerged areas)
Chaplot and Le Bissonnais	2000	experimental (field)	silty loam	2% - 8%	vertical infiltration significantly decrease with increasing slope (increase of flow velocity with slope steepness)
Janeau et al.	2003	experimental (field)	loamy	16% - 63%	vertical infiltration increase with increasing slope (the vertical component of kinetic energy has a dominant role)
Assouline and Ben-Hur	2006	experimental (laboratory)	sandy	5° - 25°	vertical infiltration increase with increasing slope (thinner crust and more rill erosion)
Chen and Young	2006	theoretical (conceptual)	-	-	normal infiltration increase (by cos(γ)) with increasing slope (only a t→0 and for equal horizontal projection length)

Essig et al.	2009	experimental (laboratory)	clay loam, loam, sandy loam	1° - 15°	vertical infiltration decrease with increasing slope (for the presence of return flow from saturated areas)
Ribolzi et al.	2011	experimental (field)	clay loam	30% - 70%	vertical infiltration increase with increasing slope (a more permeable structural crust develop under steep slopes)
Patin et al.	2012	experimental (field)	Entisol, Ultisol, Alfisol (US Taxonomy soil classification system)	10% - 110%	vertical infiltration decrease with increasing (up to 50%) slope and increase for steeper slopes (due to effect of crust formation and different land use)
Lv et al.	2013	experimental (laboratory)	sandy loam	0° - 30°	the component of flow parallel to the surface increase with slope more than the normal component
Morbidelli et al.	2015	experimental (laboratory)	loam	1° - 10°	vertical infiltration decrease with increasing slope (link between this decrease and shear stress at soil surface)
Mu et al.	2015	experimental (laboratory)	sandy loam	0° - 30°	vertical infiltration decrease with increasing slope (the surface roughness influence the slope effect)
Khan et al.	2016	experimental (laboratory)	silty loam	5° - 25°	vertical infiltration decrease with increasing slope (large effect of micro-relief features and rainfall intensity)
Morbidelli et al.	2016	experimental (laboratory)	loam (grassy)	1° - 15°	vertical infiltration decrease with increasing slope (the high surface roughness reduce the slope effect)
Wang et al.	2018	theoretical (conceptual)	-	-	vertical infiltration decrease with increasing slope (due to a ponding time prediction with a time compression approximation)