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Term premium in a fractionally cointegrated yield curve☆

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ABSTRACT

The co-movement of US Treasury yields suggests a long-run equilibrium relationship. Traditional cointegrated systems need to assume that interest rates are unit roots and thus implying non-stationary and non-mean-reverting dynamics. We postulate and estimate a fractional cointegrated model (FCVAR) which allows for mean reverting though highly persistent patterns. Our results point to the existence of such mean-reverting fractional cointegration among Treasury yields. In terms of out-of-sample forecasting, the FCVAR soundly beats the I(0) VAR model across interest rate maturities and horizons and the I(1) cointegrated VAR across maturities and short-horizons. The implied US term premium –across different maturities– proves to be quite robust across subsamples and is less volatile than the classical I(0) stationary and I(1) unit root models. Our analysis highlights the role of real factors in shaping term premium dynamics and is extended to the UK and Germany yield curves.

1. Introduction

Understanding sovereign yield curve dynamics remains a fundamental topic for investors, bankers, policy makers, media and academics. This explains why it keeps receiving so much interest across discussions in all these quarters. A specific source of term structure attention is the joint co-movement of interest rates across maturities. As Fig. 1 shows, US Treasury yields track each other quite closely despite their different maturities. Why is this the case?

Many equilibrium models, such as those based on no-arbitrage, propose the existence of common factors (level, slope and curvature) driving yield dynamics across all maturities. At the same time, researchers and policy makers have long pointed to long-rates embedding expectations of short-rates. As a result, both producing the correct short-term forecasts and capturing the common dependence of rates across maturities is of utmost importance. This is why empirical models keep trying to improve both the characterization and estimation of joint bond yield dynamics. Indeed, correctly exploiting this cross-sectional term structure co-movement has relevant economic implications for both fiscal and monetary policy, term premium identification, predictability of future macro variables as well as banking management.

Figure 1 also suggests a potential long-run dependence across the different interest rates. In the term structure literature, this behaviour has been traditionally characterized via cointegration techniques (see Campbell and Shiller, 1987 for a seminal study). In short, traditional cointegration imposes that all interest rates are unit roots or I(1) processes and that they cannot wander away from each other during long periods of time. While this methodology has advantages, such as exploiting this long-term relation across rates, this structure imposes an un-appealing non-mean reversion in rates. As explained by Campbell et al. (1997) and Diebold and Rudebusch (2013), this implies that shocks to interest

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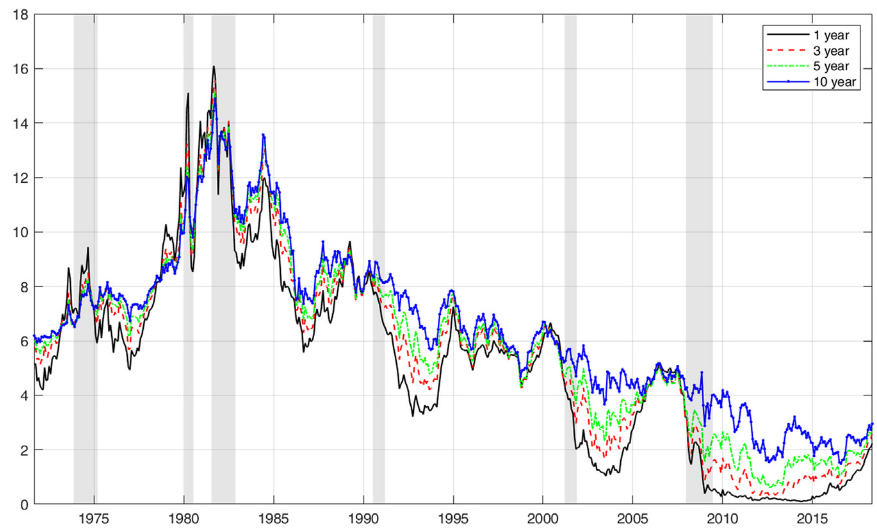

Fig. 1. US Treasury Yields. This figure plots the historical monthly series of zero-coupon US Treasury Yields (1-year, 3-year, 5-year and 10-year) from August 1971 to April 2018. Shaded areas reflect NBER recession periods.

rates have permanent effects, despite the fact that sovereign interest rates, at least in most industrialized economies, do not exhibit such behavior. Moreover, though most standard unit root methods cannot reject the presence of unit root tests individually in the interest rate series, it is well known that these methods have very low power against fractional alternatives as those used in this work (Diebold and Rudebusch, 1991; Hassler and Wolters, 1994; Lee and Schmidt, 1996, etc.).

Therefore, capturing this joint co-movement across maturities and allowing for mean-reversion dynamics seem to be desirable features in term structure models. This is what we explore and test in this paper, where we apply multivariate fractional cointegration techniques which allow for a flexible estimation of short and long-run dynamics in the term structure of interest rates. This econometric model simultaneously identifies the order of integration of rates (one, zero or a fractional number) and the potential existence of one (or several) cointegration relationships. Indeed, whether interest rates are cointegrated, fractionally cointegrated or non-cointegrated is an empirical question which we tackle in this paper. To this end, and following empirical univariate motivation of the fractional integration order of Treasury yields, we estimate a multivariate fractional cointegration vector auto-regressive (FCVAR) model (Johansen and Nielsen, 2012) with US zero-coupon yields.

We estimate the FCVAR using four interest rates capturing the short, medium and long ends of the yield curve for the US. Our findings point at a single long-run cointegration relation among the four interest rates. Our estimation results show that the common order of integration of the interest rates is 0.765 with monthly data and statistically different from zero and one. Our results thus reject modeling yields in stationary I(0) VAR and unit-root cointegration frameworks, and show that, in terms of out-of-sample forecasting, the FCVAR beats the I(0) VAR model across interest rate maturities and horizons and the I(1) cointegrated VAR across maturities and short-horizons (less than a year).

An implication of our results is that the common macro-finance shocks affecting the yield curve turn out to have transitory –rather than permanent– though long lasting effects on the term structure. Our results also reject the joint modeling of interest rates in standard stationary vector auto-regressive systems, given that we estimate the order of integration to be well and significantly above zero. Also, we find that there exists a single long-run equilibrium relationship along the term structure of interest rates. We perform separate analogous FCVAR estimations for the UK and Germany and find similar results for the yields' order of integration as well as a similar long-run cointegration relation.

Our analysis provides estimates of the term premium on long-term bonds, an important object of analysis for policy makers. Higher term premia reveal that investors require higher returns for long-term bonds, which point at a number of macro-finance or policy risks for the economy. The term premium associated with our fractional cointegrated system displays a marked degree of persistence and is clearly counter-cyclical. We analyze the sources of our term premium dynamics and show that they diverge with respect to term premia implied by stationary I(0) and unit-root I(1) models. In particular, unemployment is key to understand its counter-cyclical dynamics. Finally, we compute term premia across different maturities and show that term premia associated with longer maturities are non-linearly higher than medium-term maturities.

The paper proceeds as follows. Section 2 provides an empirical univariate motivation of the fractional integration approach for the term structure of interest rates. Section 3 summarizes the fractional cointegration econometric framework and describes the economic implications of this modeling strategy for the term structure of interest rates. Section 4 discusses the data, empirical strategy and estimation procedure. Section 5 presents the empirical results of the paper. It shows the structure of the US yield curve, the implied term premium and its economic sources –comparing to I(0) and I(1) alternatives–, and the term structure of the term premium across different maturities. It also performs out-of-sample forecasting analysis of interest rates across maturities and provides estimations for other international yield curves (UK and Germany). Section 6 concludes.

2. The yield curve order of integration: A univariate perspective

The advantage of the fractional integration framework is that it allows researchers to avoid choosing the order of integration of a variable ex-ante. If they choose the I(0) framework (Wright, 2011), they give up the possibility of meaningful cointegration relationships among interest rates. If they opt for the I(1) framework (Campbell and Shiller, 1987), they give up the relevant mean-reversion of interest rates. Moreover, these two classical settings exclude the possibility of the fractional integration order, which generalizes the integration order to include fractional orders (see Backus and Zin, 1993).

Table 1
Fractional Order of Integration of Yields, Spread and Curvature.

Yields	Integration Order	Confidence Interval
Y1	0.86	[0.80, 0.95]
Y2	0.87	[0.80, 0.94]
Y3	0.87	[0.81, 0.95]
Y4	0.88	[0.82, 0.96]
Y5	0.89	[0.83, 0.98]
Y6	0.89	[0.83, 0.97]
Y7	0.91	[0.83, 0.99]
Y8	0.90	[0.82, 0.98]
Y9	0.89	[0.82, 0.98]
Y10	0.90	[0.82, 0.99]
Y10 - Y1	0.83	[0.74, 0.94]
2*Y3 - Y10 - Y1	0.68	[0.60, 0.81]

This table shows the estimates and 95% confidence intervals of the fractional integration model for the 1-to-10-year Treasury yields employing the context of Robinson's (1994) testing procedure with the method of Bloomfield (1973), which approximates non-parametrically the ARMA part of the interest rate process.

In this section we develop some empirical univariate motivation to address these two main issues: First, what is the order of integration of interest rates? Second, do interest rates have the same integration order, lending support to a cointegration framework (either fractional or I(1))? To this end, we estimate the order of integration of the US Treasury yields (from 1 to 10 years) using the procedure by Robinson (1994), which allows for a fractional order of integration and embeds an ARMA autocorrelation pattern for the error terms following Bloomfield (1973).

Results in Table 1 show that the integration order of the Treasury yields ranges from 0.86 to 0.90, and the confidence intervals exclude both 0 and 1, although the right intervals for the longer rates are close to 1. Importantly, integration orders are not statistically different from each other since we cannot reject the null that one order is different from another.

These results provide motivation for our subsequent analysis, where we propose a fractional cointegration framework to model Treasury yields, extract the term premium of Treasury bonds and derive practical policy implications. We now turn to describe the fractional cointegration setting before moving to model estimation and results interpretation.

3. Fractional cointegration

In this section, we first briefly outline the multivariate fractional cointegration framework and lay out some of its general economic implications. Then we go on to motivate why fractional cointegration can be an appropriate modeling technique for the term structure of interest rates.

3.1. Econometric setting

Our methodology to model term structure dynamics is based on the concept of long memory behavior. In Appendix A we show some of the fundamentals of univariate fractional integration processes exhibiting long memory. Essentially, fractional integration processes allow the econometrician to capture strong levels of persistence even in the context of mean-reverting processes. Indeed, their autocorrelation function is hyperbolic, unlike the classical exponential ones of the stationary autoregressive AR (I(0)) processes. This is particularly interesting in the context of interest rates –the object of analysis in the current paper–, where researchers have devised alternative econometric techniques to capture their strong persistence, in non-stationary processes.

The natural generalization of the concept of fractional integration to the multivariate case is the idea of fractional cointegration. In this paper, we employ the Fractionally Cointegrated Vector AutoRegressive (FCVAR) model introduced by Johansen and Nielsen (2012). This method is used to determine the long-run equilibrium relationship between series. Given two real numbers d, b, the components of the vector z_t are said to be cointegrated of order d, b, denoted $z_t \sim CI(d, b)$, if all the components of z_t are $I(d)$ and there exists a vector $a \neq 0$ such that $s_t = a'z_t \sim I(d-b), b > 0$.[1] The Fractionally Cointegrated Vector AutoRegressive (FCVAR) model introduced by Johansen (2008) and further expanded by Johansen and Nielsen (2010, 2012) is a generalization of Johansen (1995) Cointegrated Vector AutoRegressive (CVAR) model which allows for fractional processes of order d that cointegrate to order $d-b$ $(b > 0)$. In order to introduce the FCVAR model, we refer first to the well-known, non-fractional, CVAR model. Let $X_t, t = 1, \dots T$ be a p-dimensional $I(1)$ time series vector. The CVAR model is:

$$\Delta X_t = \alpha^* \beta^{*\prime} X_{t-1} + \sum_{i=1}^{k} \Gamma_i^* \Delta X_{t-i} + \varepsilon_t$$
$$= \alpha^* \beta^{*\prime} L X_t + \sum_{i=1}^{k} \Gamma_i^* \Delta L^i X_t + \varepsilon_t, \quad (1)$$

where Δ refers to the first difference operator, i.e., $\Delta = (1-L)$, α^* is the vector or matrix of adjustment parameters, β^* is the vector or matrix of cointegrating vectors and the sequence of matrices Γ_i^* governs the short-run I(0) VAR dynamics. The simplest way to derive the FCVAR model is to replace the difference and lag operators Δ and L in (1) by their fractional counterparts, Δ^b and $L_b = 1 - \Delta^b$, respectively. We then obtain:

$$\Delta^b X_t = \alpha \beta' L_b X_t + \sum_{i=1}^{k} \Gamma_i \Delta^b L_b^i X_t + \varepsilon_t, \quad (2)$$

which is applied to $X_t = \Delta^{d-b}(Y_t - \mu)$, where Y_t is the $p \times 1$ vector of our series of interest and μ is a level parameter vector which accommodates a non-zero starting point for the first observation on the process.[2] α, β, Γ have an analogous interpretation to $\alpha^*, \beta^*, \Gamma^*$ in (1) but they only coincide under $d = b = 1$. We therefore have that:

$$\Delta^d (Y_t - \mu) = \alpha \beta' L_b \Delta^{d-b} (Y_t - \mu) + \sum_{i=1}^{k} \Gamma_i \Delta^d L_b^i (Y_t - \mu) + \varepsilon_t, \quad (3)$$

where ε_t is p-dimensional independent and identically distributed with mean zero and covariance matrix Ω. The parameters have the usual interpretations known from the CVAR model. In particular, α and β are $p \times r$ matrices, where $0 \leq r \leq p$. The columns of β are the cointegrating relationships in the system, that is to say the long-run equilibria. The parameters Γ_i govern the short-run behavior of the variables –with k being the lag length of the VAR– and the coefficients in α represent the speed of adjustment towards equilibrium for each of the variables. Thus, the FCVAR model permits simultaneous modelling of the long-run equilibria, the adjustment responses to deviations from the equilibria and the short-run dynamics of the system. Notice that the cointegration intuition in the fractional case is analogous to the I(1) case, i.e. that there is a long-run relation among the variables. In Johansen and Nielsen (2012) and Nielsen and Popiel (2016) one can find estimation and inference explanations of the model, and the latter provides Matlab computer programs for the calculation of estimators and test statistics.

[1] A more general definition of fractional cointegration allows different orders of integration for each individual series. See, e.g., Robinson and Marinucci (2001), Robinson and Hualde (2003) and others.

[2] The use of a long-run mean term μ in the FCVAR model and its interaction with the initial values is not yet well established in the literature. Recent papers discussing this issue are Johansen and Nielsen (2016) and Nielsen and Shibaev (2018).

3.2. Why an FCVAR model for the yield curve?

We now discuss several features of the FCVAR to model yield curve dynamics. The first one is model generality and flexibility. The FCVAR model lets the data at the same time (i) determine the order of integration of interest rates –without having to resort to unit root or fractional integration pre-testing on the order of integration ex-ante (with the well-known lack of statistical power of unit root tests)– and (ii) estimate the joint multivariate model dynamics allowing for the combination of short and long-memory dynamics. In particular, the FCVAR can accommodate fractional integration. Empirically, several authors have shown that interest rates tend to display significant fractional integration dynamics, as found in Backus and Zin (1993), Gil-Alana and Moreno (2011), Osterrieder (2013), Abbritti et al. (2016) and Golinski and Zaffaroni (2016), among many others. Theoretically, and inspired by the work of Robinson (1978) and Granger (1980), Altissimo and Zaffaroni (2009) show that aggregation of sub-indices can explain inflation persistence. If aggregation explains fractional integration in inflation, then interest rates can all inherit fractional integration due to standard inflation targeting strategies by monetary policy makers.

Second, the proposed FCVAR model also allows for explicit long-run relations among the yields since it endogenously estimates the (potential) cointegration relationships among the yields. I(1)-cointegration has been proposed in the term structure literature by several authors, such as Campbell and Shiller (1987), Cieslak and Povala (2015) and Bauer and Rudebusch (2020). The advantage of our model is that the FCVAR accommodates cointegration without the need to assume I(1) dynamics for interest rates, since interest rates can be cointegrated and individually mean-reverting. In sum, given the separate evidence of fractional integration for yields and cointegration, it seems sensible to feature them jointly in order to capture the correct joint yield dynamics.

Third, by identifying the actual integration order of the variables and the potential cointegration relations, it avoids potentially important mis-specifications in key policy objects, such as the term premium, as we show in the paper. In particular, letting the data choose the order of integration represents an important advantage, avoiding the risk of over/under-differencing the variables. As shown by Cochrane and Piazzesi (2008), by assuming I(1) cointegration or an I(0) VAR model, we may be mis-specifying the model estimates, parameters, test restrictions and implied dynamics, such as the term premium. This has become a very relevant issue in recent times, as both academics and policy makers strive to understand the effects of quantitative easing policies (and subsequent tapering) on term premium dynamics (see Yellen, 2017, among others).

One limitation of the FCVAR model is that it is a model for individual interest rates (4 in our case, as we show below) and not for the entire yield curve. It is thus different from popular models, such as the affine term structure models which model the full yield curve. Despite this fact, our model includes information on the short, medium and long ends of the yield curve, thus incorporating a wealth of macro-finance information to identify the expectations of the short-rate and thus the term premium. By including the cointegration relation, it accounts for long-run co-movements across interest rates, providing an interesting economic interpretation. Moreover, as shown below, the FCVAR out-of-sample forecasts of interest rates are better than the I(0)VAR (across maturities and forecast horizons) and the I(1) CVAR models (across maturities and short forecast horizons).

Of course, there are other alternative interesting techniques for modeling interest rates. One of them is regime switching (see Ang and Bekaert, 2002 and Baele et al., 2015, among others). Regime switching has the appealing feature of allowing shifts in meaningful key reduced-form or policy parameters, such as the reaction to inflation deviations from target or changes in interest rate inertia induced by financial stability purposes. These shifts influence the whole term structure, thus shaping joint yield dynamics. While the fractional cointegration approach does not model these parameter shifts, it can be consistent with regime switching dynamics. Indeed, as explained by Diebold and Inoue (2001), the dynamics of fractional integration and regime switching are easily confused, with fractional integration being able to capture some of the embedded autocorrelations derived from regime switching processes.

Another interesting modeling alternative for interest rate is the one based on “near-cointegration” proposed in Jardet et al. (2013). They propose a no-arbitrage term structure model that takes into account the persistence of the variables (short-rate, the spread between the long and the short rates, and GDP growth) by using a “near cointegration” approach. Using this method, they still impose integer degrees of differentiation, not taking into account the possibility of fractional values, unlike in the present work.

4. Data and estimation

In our empirical work, we employ monthly series corresponding to the U.S. Treasury Yield Curve. The data was obtained online from the work by Gürkaynak et al. (2007). Their yield curve estimates are updated periodically and provide a benchmark US Treasury zero-coupon yield curve. In our extension to other countries, we use publicly available yields from Germany and the UK. Our baseline specification includes four series, namely the one, three, five and ten year Treasury yields. In this way, our data vector Y_t includes information about the short, medium and long end of the yield curve. By including different parts of the term structure, our model captures key macro-finance information, including future economic and financial expectations. Our dataset covers observations from August 1971 up to April 2018. Figure 1 shows the dynamics of the four interest rates for our sample period.

In terms of estimation, we proceed as follows: We first assume that a sample of length $T+N$ is available on Y_t, where N denotes the number of observations used for conditioning. As shown in Johansen and Nielsen (2012), model (3) can be estimated by conditional maximum likelihood, conditioned on N initial values, by maximizing the following function:

$$\log L_T(\lambda) = -\frac{T}{2}(\log(2\pi)+1) - \frac{T}{2}\log\det\left\{T^{-1}\sum_{t=N+1}^{T+N}\varepsilon_t(\lambda)\varepsilon_t(\lambda)'\right\}. \tag{4}$$

For model (3) the residuals are:

$$\varepsilon_t(\lambda) = \Delta^d(Y_t-\mu) - \alpha\beta'\Delta^{d-b}L_b(Y_t-\mu) - \sum_{i=1}^{k}\Gamma_i\Delta^d L_b^i(Y_t-\mu), \tag{5}$$

with $\lambda = (d, b, \mu', \text{vec}(\alpha)', \text{vec}(\beta)', \text{vec}(\Gamma_1)' \ldots \text{vec}(\Gamma_k)')'$. It is shown in Johansen and Nielsen (2012) and Dolatabadi et al. (2016) that, for fixed (d, b), the estimation of model (3) is carried out as in Johansen (1995). In this way the parameters $\mu, \alpha, \beta, \Gamma_1 \ldots \Gamma_k$ can be concentrated out of the likelihood function. Then we only need to optimize the profile likelihood function over the two fractional parameters, d and b. As explained by Johansen and Nielsen (2018), the likelihood ratio test of the usual CVAR is asymptotically $\chi^2(2)$ and the likelihood ratio test of the hypothesis that $d=b$ in the fractional model is asymptotically $\chi^2(1)$. Hence these tests are very easy to implement and can be calculated using the software package of Nielsen and Popiel (2016)

5. Empirical results

In this section we present and discuss the first empirical results of the paper. We first show the estimates of the FCVAR model and discuss the long-run dynamic implications for yields. We then extract the FCVAR-forward term premium, compare it with alternative I(0) and I(1) counterparts and provide an interpretation of its underlying economic sources. We also derive results on the implied term structure of yield term premia and provide subsample analysis. In a subsequent out-of-sample analysis, we show that our FCVAR model outperforms the I(0) VAR model at all horizons. It also outperforms the CVAR when the forecast horizon is approximately below 1 year, while it does equally well for longer horizons. In the last subsection, we estimate the FCVAR and obtain the implied forward term premia for two additional yield curves: UK and Germany.

5.1. Baseline estimates

5.1.1. FCVAR orders of integration

The dataset in Gürkaynak et al. (2007) provides daily data of Treasury yields from maturities 1-year to 30 years. To capture some relevant maturities at the short, medium and long end of the yield curve, we work with the 1-year ($i_t^{(12)}$), 3-year ($i_t^{(36)}$), 5-year ($i_t^{(60)}$) and 10-year ($i_t^{(120)}$) US Treasury yields.[3] We work with the monthly frequency, as results can then be related to key macro variables, such as unemployment, consumer inflation or industrial production. We use end-of-the-month interest rate observations over each month to construct the monthly dataset, which spans the August 1971-April 2018 sample period.

We proceed as follows. We first run the FCVAR unrestricted and then estimate the model assuming that $d = b$. This second model is a relevant one, as it implies that the cointegration residual is the classical I(0).When we run the unrestricted FCVAR system, we obtain the following estimated model:

$$\Delta^{0.756}(Y_t - \mu) = \alpha\beta' L_{1.184}\Delta^{0.756-1.184}(Y_t - \mu) + \sum_{i=1}^{k}\Gamma_i \Delta^{0.756} L_{1.184}^{i}(Y_t - \mu) + \varepsilon_t. \quad (6)$$

Results are based on a VAR(1) for short-run dynamics ($k = 1$), as selected by the Hannan-Quinn criterion.[4] The parameter d is estimated to be 0.756 (with standard deviation 0.034) and the parameter b is estimated to be 1.184 (with standard deviation 0.087).[5] This implies that the error term displays anti-persistence, being therefore stationary and with the shocks reverting more often than those expected from a random series. When we impose that $d = b$, we obtain the following results:

$$\Delta^{0.765}(Y_t - \mu) = \alpha\beta' L_{0.765}(Y_t - \mu) + \sum_{i=1}^{k}\Gamma_i \Delta^{0.765} L_{0.765}^{i}(Y_t - \mu) + \varepsilon_t. \quad (7)$$

Table 2
LR Test, US Treasury Yields, $d = b$ v/s $d \neq b$.

Unrestricted log-like:	1361.136
Restricted log-like:	1357.200
LR statistic:	7.873
p-value (Bootstrap):	0.183

This table shows the bootstrap results of the Likelihood Ratio (LR) Test, testing the likelihood of the FCVAR model with d different from b and the FCVAR model with the restricted model where $d = b$.

Table 3
LR Test, US Treasury Yields, CVAR v/s FCVAR.

Unrestricted log-like:	1357.200
Restricted log-like:	1345.663
LR statistic:	23.073
p-value (Bootstrap):	0.003

This table shows the bootstrap results of the Likelihood Ratio (LR) Test, testing the likelihood of the FCVAR model vis à vis the I(1) CVAR model.

Table 4
Cointegrating Rank Test, US Treasury Yields.

Rank	Log-Likelihood	LR statistic
0	1343.603	60.193
1	**1361.136**	**25.127**
2	1371.076	5.248
3	1372.619	2.162
4	1373.700	–-

This table shows the results of the cointegrating rank test for the FCVAR model. In bold, the selected cointegration rank.

The estimates imply a unique fractional cointegration relation with d=0.765 –very similar to our benchmark 0.756– and a standard deviation of 0.050, with a 95% confidence interval including the set (0.688, 0.824). Given the similarity of the estimates, we now assess statistically the restriction $d = b$. A likelihood ratio (LR) test comparing the likelihoods of the unrestricted model ($d \neq b$) and the restricted model ($d = b$), has a p-value of 0.014. However, it could be the case that this test overejects due to size distortions. Hence, we run a bootstrap simulation of the likelihood ratio test. When we run this small-sample study, we indeed find that one cannot reject that $d = b$ (p-value=0.183) –see Table 2–. Table 3 also shows the results of a small-sample LR test which reveals that the I(1) CVAR is rejected in favor of the FCVAR. Hence, throughout the paper, we show the results implied by the model under $d = b$. We note however, that subsequent results turn out quite similar under both specifications.

Table 4 reports the cointegrating rank test –analogous to a cointegration test– and identifies a single long-run cointegration relation for interest rates. In turn, the alternative of not having a cointegrating relationship is clearly rejected. Hence, the FCVAR model is validated. Finally, the level parameter μ (with associated standard errors in parentheses) is estimated at:

$$\hat{\mu} = \begin{bmatrix} 5.254 & 5.769 & 6.010 & 6.123 \\ (0.440) & (0.377) & (0.341) & (0.308) \end{bmatrix}'$$

5.1.2. FCVAR long-run analysis

The estimated single long-run fractional cointegration vector for the US term structure implies that $\hat{\beta} = [1.000, \; -3.314, \; 3.240, \; -0.903]'$, where the elements of this vector are associated with the 1, 3, 5 and 10-year bond rates, respectively. Thus, while loadings on the medium end of the yield curve are more than twice higher than those in the short and long ends, the sum of the four loadings is close to zero. Our estimates thus imply the existence of three stochastic trends and one cointegration relationship. While there are alternative underlying models

[3] By using the 1-year interest rate as the short-term interest rate, our term premium is an approximation to the true term premium, as the riskless rate is the 1-month interest rate.

[4] Other likelihood criteria, such as AIC and BIC, produced the same result. This is a relevant issue noting that the FCVAR model can suffer from identification issues when the number of lags is unknown (see Carlini and Santucci de Magistris, 2017).

[5] This large number can be a consequence of the conditioning on few initial values, although the conclusions of our study seem not to be affected by this large number.

consistent with our results, Appendix B shows an example where each of the three yields loads on a different set of stochastic trends. While one of the stochastic trend affects all yields, the different combinations of stochastic trends make these three yields display three different stochastic trends themselves. The fourth yield, in turn, is a linear combination of the other three yields and this combination is precisely the long-run cointegration relation among the yields. The finding of this single long-run cointegration vector –together with this model setting– is consistent with the existence of an underlying stochastic trend affecting all yields and triggering long-run parallel shifts in the yield curve –similar to the classical level factor in the term structure literature–. Additionally, this model accommodates two alternative underlying stochastic trends which can cause permanent shifts in the yield curve slope and curvature. This is consistent with our univariate estimation of the fractional integration order of the spread and the curvature (see last two rows of Table 1), which are estimated to be 0.83 and 0.68, both statistically different from 0 and 1.

The corresponding speed of adjustment vector is estimated at:

$$\hat{\alpha} = \begin{bmatrix} 0.111 & 0.229 & 0.253 & 0.283 \\ (0.102) & (0.089) & (0.083) & (0.075) \end{bmatrix}'$$

As a result, the implied speed of adjustment with respect to deviations from the long-run relationship is the fastest (and statistically significant, given the standard deviation) for the 10-year rate. In contrast, the 1-year rate adjustment to deviations from this fractional cointegration is very sticky, almost null (and statistically non-significant, given the standard deviation). The short-rate thus tends to be less driven by the long-run relation among rates and more influenced by its own short-run dynamics, at least at high frequencies. So, shocks affecting specifically the medium and long-end of the yield curve –and which generate deviations from the long-run relationship– are transmitted to the short-rate very slowly, while specific shocks affecting the short and medium ends of the yield curve –and, again, to the extent that they generate deviations from the long-run relationship– are transmitted to the 10-year rate relatively fast.

5.1.3. Robustness analysis: Integration order across yields

By construction, one restriction of the FCVAR model is that it constrains all the Treasury yields to have the exact same integration order. One may argue that yields could have different integration orders and thus should not be cointegrated. We entertain this possibility and estimate an alternative Fractional VAR model, which allows for different integration orders across yields. In particular, we estimate a VARFIMA(1,D,0), i.e. allowing for a first order VAR without any MA part, and fractional dynamics (D is a 4×4 diagonal matrix where each entry includes the (potentially) fractional order of integration of each yield). Our estimates imply that the orders of integration are (with standard errors in parentheses) 0.771 (0.076), 0.783 (0.065), 0.799 (0.061) and 0.823 (0.059) for the 1, 3, 5 and 10-year yields, respectively. We cannot statistically reject that the parameters are the same, thus lending support to the FCVAR specification. Moreover, neither of these values is significantly different from the FCVAR estimate of the common integration order, which is 0.765.

5.2. Term premium analysis

Once we have determined that Treasury yields are fractionally cointegrated, we can examine the implied forward term premium. Following Wright (2011), we compute the forward term premium as the model-implied five-to-ten-year forward rate minus the average expected one-year interest rate from five to ten years hence:

Table 5
Forward Term Premium, Risk Neutral Rate: Descriptive Statistics.

	Variable	Mean	St.dev.
I(0)-VAR	Risk neutral rate	5.011	1.351
	Forward Term Premium	2.030	1.741
I(1)-CVAR	Risk neutral rate	5.212	3.628
	Forward Term Premium	1.802	1.750
FCVAR	Risk neutral rate	5.366	2.964
	Forward Term Premium	1.648	1.190

This table shows the mean and standard deviation of the forward term premium and risk neutral rates implied by the three alternative term structure models.

$$ftp_t = f_t^{(120-60)} - \frac{1}{5} E_t \sum_{j=5}^{9} i_{t+12j}^{(12)}. \quad (8)$$

Based on the estimates of our FCVAR model, we can identify the implied baseline forward term premium (ftp_t).[6] This is plotted in Fig. 2. As the figure shows, the implied forward term premium is markedly counter-cyclical and no clear trend emerges. While the forward term premium is positive during most of the sample period, it also displays low negative values at the end of the 70s and beginning of the 80s (reaching values around -0.5%). During the recent 2008 financial crisis, the forward term premium also increased to values higher than 3%, but it has declined since then, with forward term premium levels below 1% by the end of the sample.

Table 5 shows the mean and standard deviation of the forward term premia and risk-neutral rates implied by the I(0)-VAR, I(1)-CVAR and FCVAR models, respectively. The I(0)-VAR model generates the least variable risk neutral rate, due to the fast mean reversion of forward-looking expectations. The opposite is the case for the CVAR model, where expectations are the most volatile. The FCVAR model is the one which clearly delivers the most stable forward term premium in terms of standard deviation (one third lower than the CVAR and I(0)-VAR counterparts). Its mean is also the lowest, 20 and 40 basis points lower than the CVAR and I(0)-VAR models, respectively. Table 6 shows the correlation of the forward term premia and risk-neutral rates with four macro variables: Federal Funds rate, unemployment, industrial production growth and the forward term premium itself. While the FCVAR and the CVAR forward term premia display a negative correlation with the Federal Funds rate, this correlation is positive for the I(0)-VAR. Also, the risk-neutral rates implied by the FCVAR and the CVAR have a negative correlation with their respective forward term premia, whereas the opposite is the case for the I(0)-VAR. All term premia have a positive correlation with unemployment, a theme we revisit below.

The top graph in Fig. 3 plots the forward term premia implied by the following three models: FCVAR, I(0) VAR and I(1) CVAR. It shows how the I(0)-implied forward term premium is substantially higher during the early 80s and becomes increasingly negative since 2015. The differences are quite sizable during some periods. The I(0)-implied forward term premium is higher than the FCVAR-term premium from 1976 to 1995 (reaching a difference of almost 4% in the early 80s). This gap exhibits a downward trend, revealing the downward trend in the I(0) implied forward term premium during the first part of the sample. The downward trend in the I(0)-implied forward term premium thus reveals the challenge that I(0) models face when describing the true counter-cyclical nature of the forward term premium (see, e.g., Bauer et al., 2012 small-

[6] The forward term premium (as well as the yield term premium defined below) embeds a convexity term, given that our model is homoscedastic. This convexity term, which depends on the maturity of the term premium, is constant and therefore does not affect the term premium dynamics.

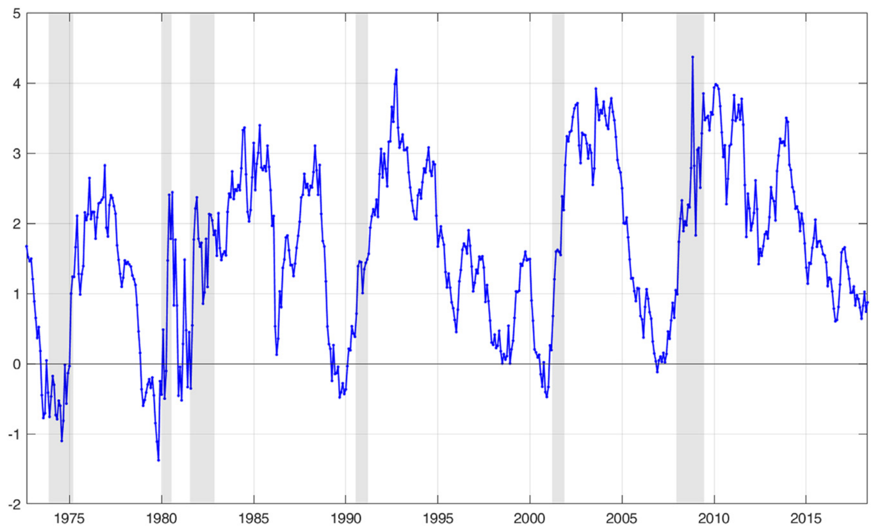

Fig. 2. Forward Term Premium FCVAR. This figure plots the monthly 10-5 forward term premium implied by the FCVAR. Shaded areas reflect NBER recession periods.

Table 6
Forward Term Premium, Risk Neutral Rate: Descriptive Statistics.

	Variable	Corr w/FFR	Corr w/ftp	Corr w/unempl	Corr w/Δ Y
I(0)-VAR	Risk neutral rate	0.936	0.503	-0.024	0.190
	Forward Term Premium	0.564	1	0.578	-0.059
I(1)-CVAR	Risk neutral rate	0.981	-0.709	0.096	0.106
	Forward Term Premium	-0.749	1	0.358	-0.132
FCVAR	Risk neutral rate	0.968	-0.422	0.134	0.069
	Forward Term Premium	-0.524	1	0.484	-0.042

This table shows the correlations of the three alternative forward term premia and risk neutral rates with several macro-finance variables: Federal Funds Rate, forward term premium, unemployment and industrial production growth, respectively.

sample analysis of I(0)-type models). In contrast to the I(0)-implied model, the CVAR-implied forward term premium is lower than the FCVAR-implied one during most of the first 15 years of the sample. This difference reaches its maximum value in the last years of the 70s and the first years of the 80s (reaching beyond -3%), when Treasury yields were especially volatile due to monetary policy tightening in an era of high inflation rates.

By the last years of the sample –at the time of policy rates close to the zero lower bound–, important differences remain and they have different signs depending on the model at hand: Around 0.5% higher in the CVAR and around 2% lower in the I(0) VAR. The first column of Fig. 4 examines the patterns in the 1-year rate expectations for the three models during the post-2006 period for three alternative horizons (1-year, 5-year and 10-year). Differences

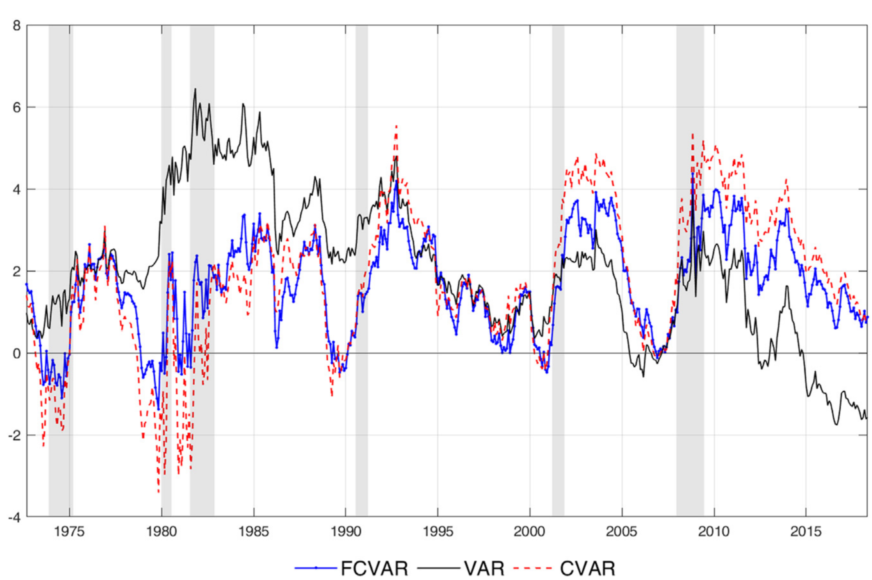

Fig. 3. Differences in Forward Term Premium: I(0) and CVAR v/s FCVAR. This figure plots the three monthly 10-5 forward term premia (CVAR, FCVAR and I(0)-VAR). Shaded areas reflect NBER recession periods.

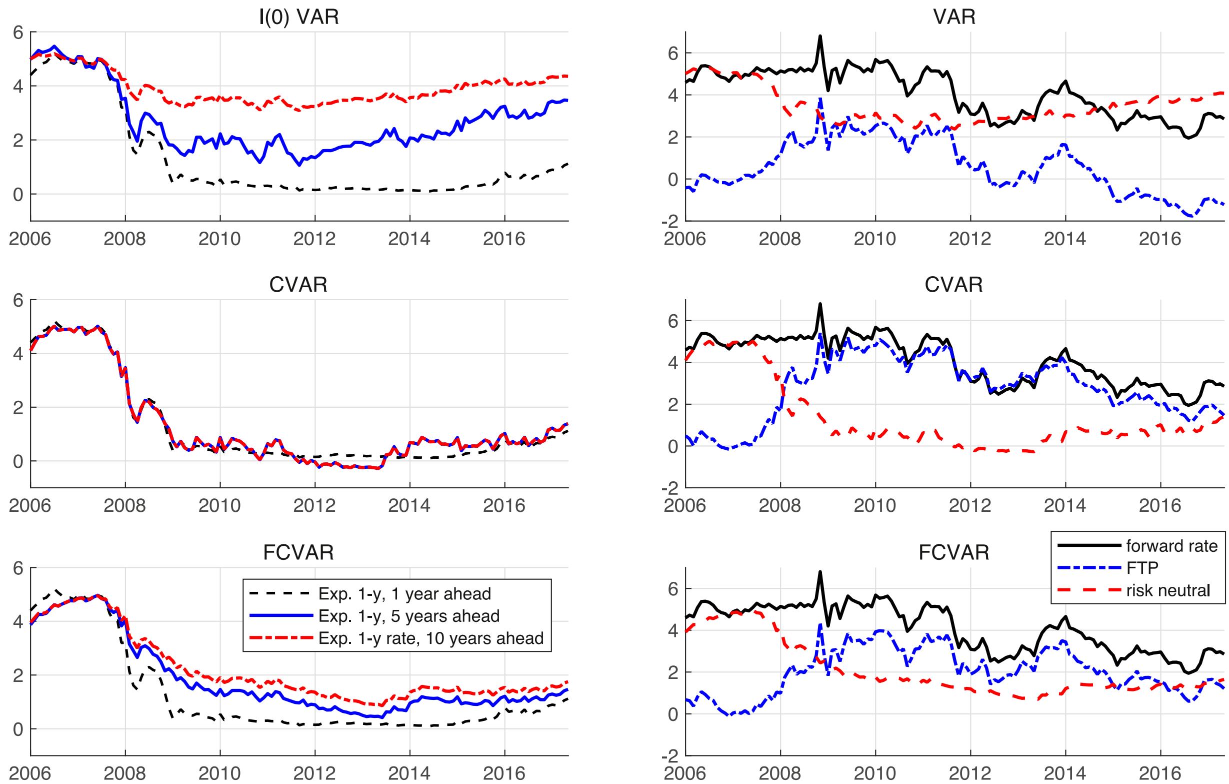

Fig. 4. Differences in Long-Term Expectations, Forward Term Premium and Risk-Neutral Rates: I(0) and CVAR v/s FCVAR. The graphs in the left column show the long-term expectations of the 1-year rate implied by the three models (I(0)-VAR, I(1)-CVAR and FCVAR (I(d))) for different horizons (1-year, 5-years and 10-years ahead), whereas those in the right column plots the implied forward term premium and risk-neutral rates across the three models together with the 10-5-year forward rate.

are striking. While the I(0) model produces long-run (10-year) expectations above 4% (close to the full sample average) –implying a negative term premium by the end of the sample (see top graph in the second column of Fig. 4)–, the opposite is the case for the I(1)-CVAR, where implied expectations are very close to zero (in fact they are negative during almost three years!). The FCVAR-implied long-run expectations are between 1 and 2%, showing a realistic slow mean reversion in the context of a slow economic recovery. In sum, our FCVAR-identified forward term premium is less volatile than its I(0) and CVAR counterparts. Our analysis shows that this is due to the fact that the I(0) model impinges too little volatility to the risk-neutral rate, whereas the CVAR imparts too much volatility.

Theoretical and empirical research identifies two main reasons behind an increase in term premia: On the one hand, an increase in inflation uncertainty (see, e.g., Wright, 2011) and on the other an increase in economic risk (see e.g. Bauer et al., 2012). Thus, a correct identification of the term premium is crucial for an understanding of the economic forces behind term premium dynamics as well as for the appropriate policy response. In fact, these two risk factors call for opposite monetary policy response: Central banks should increase interest rates if increasing risk premia reflect inflation uncertainty, while they should reduce them when a spike in term premia reflects economic and financial risk (see related comments in Bernanke, 2006). It is therefore quite important to understand which one is likely to be the dominating factor behind eventual term premium increases.

To shed some light on this issue, we follow, e.g, Backus and Wright (2007), Gagnon et al. (2011), and Wright (2011), which introduce the following ordinary least squares regression model so as to explain historical time variation in the forward term premium:

$$ftp_t = \alpha + \beta x_t + \eta_t, \tag{9}$$

Table 7
Forward Term Premium Drivers: Multiple Regression Model.

1990m4-2018m4	FTP I(0)	FTP I(1)	FTP FCVAR
Constant	-5.87***	-0.47	-0.56
	(0.91)	(0.58)	(0.47)
Long-run Inflation Disagreement	1.87***	-0.58***	-0.25
	(0.28)	(0.21)	(0.17)
Unemployment Rate	0.25***	0.77***	0.54***
	(0.07)	(0.06)	(0.06)
Recession dummy	0.29	0.63**	0.08
	(0.25)	(0.28)	(0.17)
Adj. R2	0.45	0.59	0.50

This table shows the result of the simple OLS regressions of the alternative forward term premium on macro-finance variables. Newey-West-corrected standard errors appear in parentheses.

where ftp_t is a measure of the forward term premium –I(0)-VAR, FCVAR or CVAR–, x_t denotes a vector of regressors and η_t is the error term. In practice, we will consider two models. In the first model, which is very similar to the ones in Backus and Wright (2007), Wright (2011) and Bauer et al. (2012), we regress the forward term premium on measures of inflation uncertainty and real economic activity. Specifically, we measure inflation uncertainty with the long-run inflation disagreement series measured by the Michigan Survey of Consumers, which captures the interquartile range of five-to ten-year-ahead inflation expectations. Business cycle uncertainty is captured with the unemployment rate and an NBER recession dummy.

We compare in Table 7 the results obtained with the I(0)-VAR TP, the CVAR and the FCVAR term premium. The dimension of the full sample, which starts in April 1990 and finishes in April 2018, is constrained by the availability of the long-run inflation disagreement series. We find that the correct identification of the persistence of the forward term premium has a strong influence in their interpretation. While the stationary I(0) forward term premium is strongly positively related to inflation uncertainty, the opposite is the case for the unit root-I(1) forward term premium, as the conditional correlation with inflation uncertainty is negative. In contrast, the results of the FCVAR model show no evidence of correlation with our measure of inflation uncertainty.

The differences between the results obtained with the stationary I(0) model and the ones of the I(1) and FCVAR alternatives are due to the implied volatility of the risk neutral rate. As explained by Bauer et al. (2012), the stationary I(0) model implies too fast mean reversion of expected interest rates and too little volatility of the risk neutral rate. As a consequence, the term premium identified with the I(0) model inherits the trend of nominal interest rates, which is in turn related to the downward inflation trend. In the I(1) and FCVAR specifications, instead, the risk neutral rates respond much more to changes in the short rate, and the implied term premium does not inherit the downward trend of nominal interest rates and inflation. Interestingly, the recession dummy is significant in the I(1) model but not in the FCVAR model.

To control for the correlation between unemployment and inflation uncertainty, we correct the baseline regression by substituting the unemployment variable for the residuals of the unemployment regression on a constant and long-run inflation disagreement. Table 8 shows the associated results. In this instance, long-run inflation disagreement enters also significantly in the term premium regressions for the FCVAR and CVAR models, but this relationship is weaker and less robust, and appears to be dominated by the strong relationship between term premium and unemployment rates. Overall, our results are in line with the findings of Bauer et al. (2012), who claim that a more precise estimation of the persistence of interest rates is crucial to avoid too fast mean reversion of expected interest rates and thus an underestimation of risk neutral rates. While these regression specifications are admittedly very simple, their insights carry over to regression models with additional controls considered by Gagnon et al. (2011) (core PCE inflation, interest rate volatility and the Economic Policy Uncertainty index of Baker et al., 2016).

Table 8
Forward Term Premium Drivers: Alternative Model.

1990m4-2017m12	FTP I(0)	FTP I(1)	FTP FCVAR
Constant	-5.56***	0.46	0.08
	(0.89)	(0.57)	(0.46)
Long-run Inflation Disagreement	2.28***	0.65***	0.61**
	(0.30)	(0.19)	(0.15)
Unemployment Residual	0.25***	0.77***	0.54***
	(0.07)	(0.06)	(0.06)
Recession dummy	0.29	0.63**	0.08
	(0.25)	(0.28)	(0.19)
Adj. R2	0.45	0.59	0.50

This table shows the result of the OLS regressions of the alternative forward term premium on macro-finance variables. The Unemployment Residual variable is the residual of the unemployment rate regression on a constant and long-run inflation disagreement. Newey-West-corrected standard errors appear in parentheses.

We now turn to analyze the term structure of the term premia. For our baseline analysis, we have chosen a standard forward term premium (the difference between the five-to-ten-year forward rate and the average of the five expected 1-year rates 5 years hence), but we can alternatively compute the three following yield term premia based on a standard decomposition:

$$ytp_t^{(n)} = i_t^{(12n)} - \frac{1}{n} E_t \sum_{j=0}^{n-1} i_{t+12j}^{(12)}, \qquad (10)$$

where n can be equal to 3, 5 or 10 years. Figure 5 shows these three yield term premia together with the baseline one. Results are quite sensible and reveal three key features. First, the yield term premia are highly correlated and clearly counter-cyclical. Second, yield term premia associated with longer maturities are higher than shorter maturities. Third, differences across yield term premia are clearly non-linear: They become larger at high values across term premia.

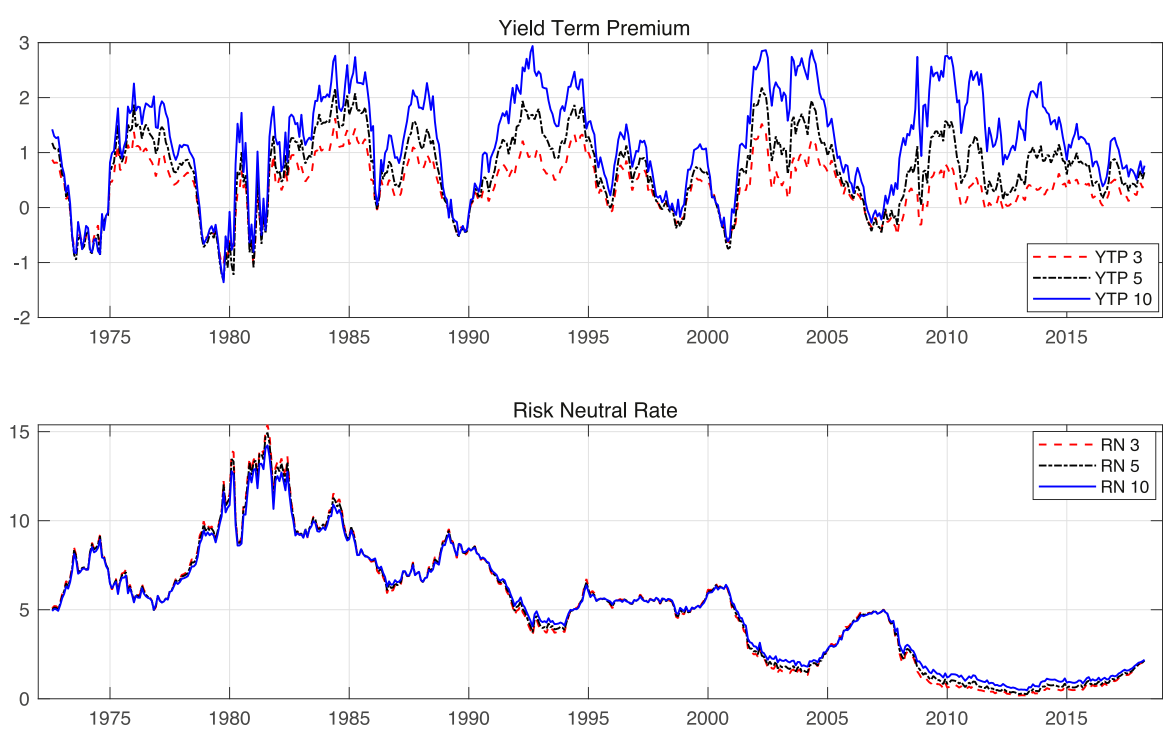

Fig. 5. Term Structure of Yield Term Premia: FCVAR. The top figure shows the FCVAR-implied yield term premia computed at different maturities, ie. the 3, 5 and 10-year yield term premia following equation (10). The bottom figure plots the associated risk-neutral rates.

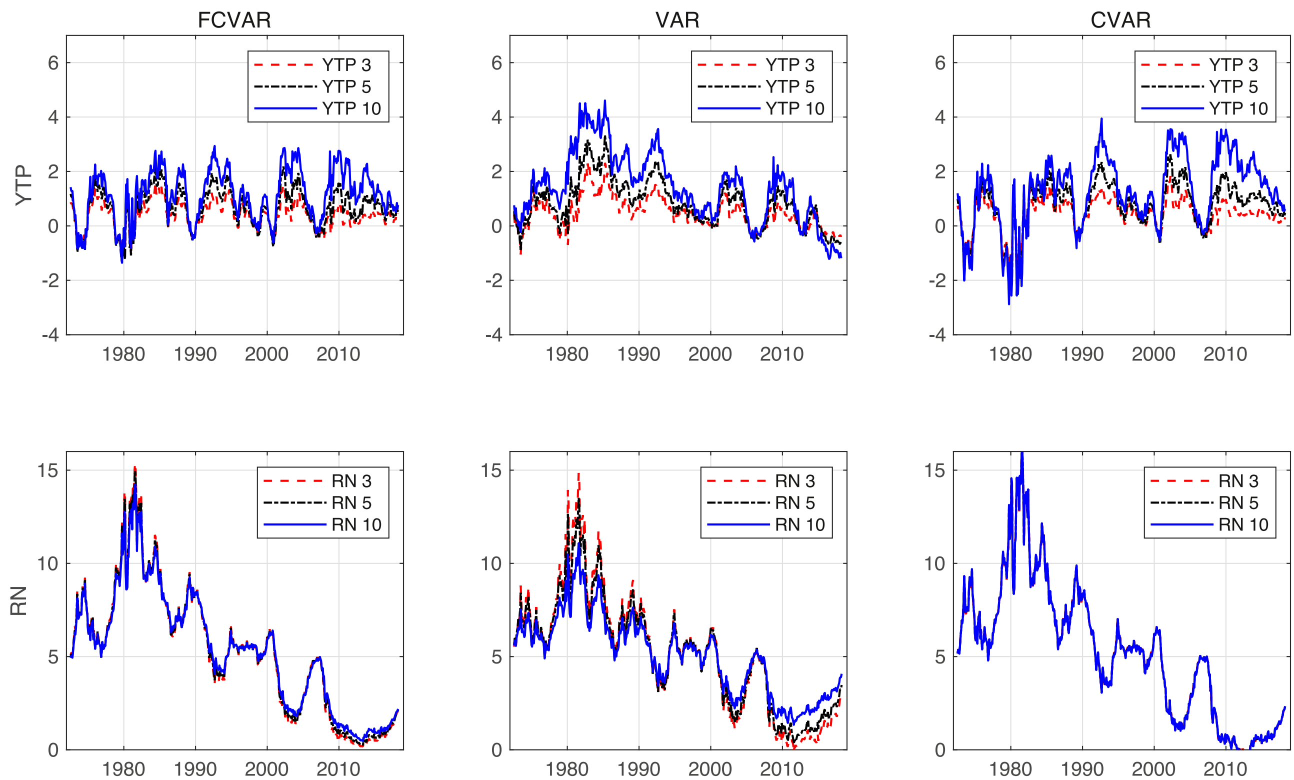

Fig. 6. Term Structure of Yield Term Premia: VAR, FCVAR, CVAR. The top figure shows the term structure of yield term premia for the (from left to right) FCVAR, I(0)VAR and I(1) CVAR models. The yield term premia are computed at different maturities, ie. the 3, 5 and 10-year yield term premia following equation (10). The bottom figure plots the associated risk-neutral rates.

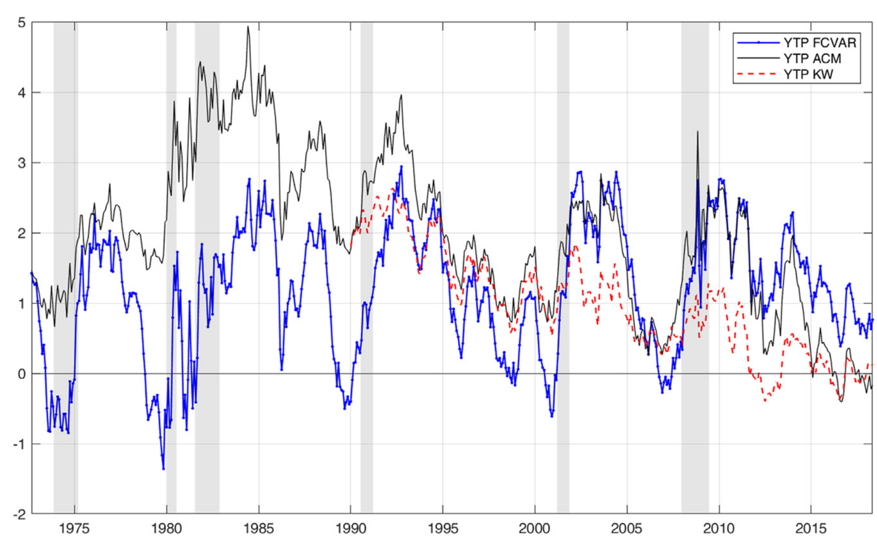

Fig. 7. Yield Term Premium Comparison Across Models. This Figure compares our FCVAR-implied yield term premium with those by Kim and Wright (2005) (YTP KW) and Adrian et al. (2013) (YTP ACM). All term premia are 10-year yield term premia. Shaded areas reflect NBER recession periods.

We shed further light on the term structure of yield term premia by graphing the 3 term premia for the 3 models (FCVAR, I(1)-CVAR, I(0) VAR), together with the associated risk-neutral rates. Results are shown in Fig. 6. Overall, the yield term premia estimated by the FCVAR display less volatility than the other 2 sets of counterparts. The I(0) VAR-implied yield term premia all display a clear downward trend, with very large differences across yield premia during some periods, such as the 1980s. The I(1) CVAR-implied yield term premia display counter-cyclical dynamics but take unrealistic negative values in the 1970s.

Figure 7 compares our FCVAR-implied yield term premium with those by Kim and Wright (2005) and Adrian et al. (2013). This comparison is done for the 10-year term premium (YTP10), which is available from these authors. The figure shows that our implied term premium presents a clearer countercyclical pattern than the other two term premia. These alternative term premia display a clear downward trend starting in the early 80s and continuing in the 90s (this is also the case in the term premium derived by Wright, 2011). This is due to the fact that these two term premia are based on an I(0) model for the factors –which we reject

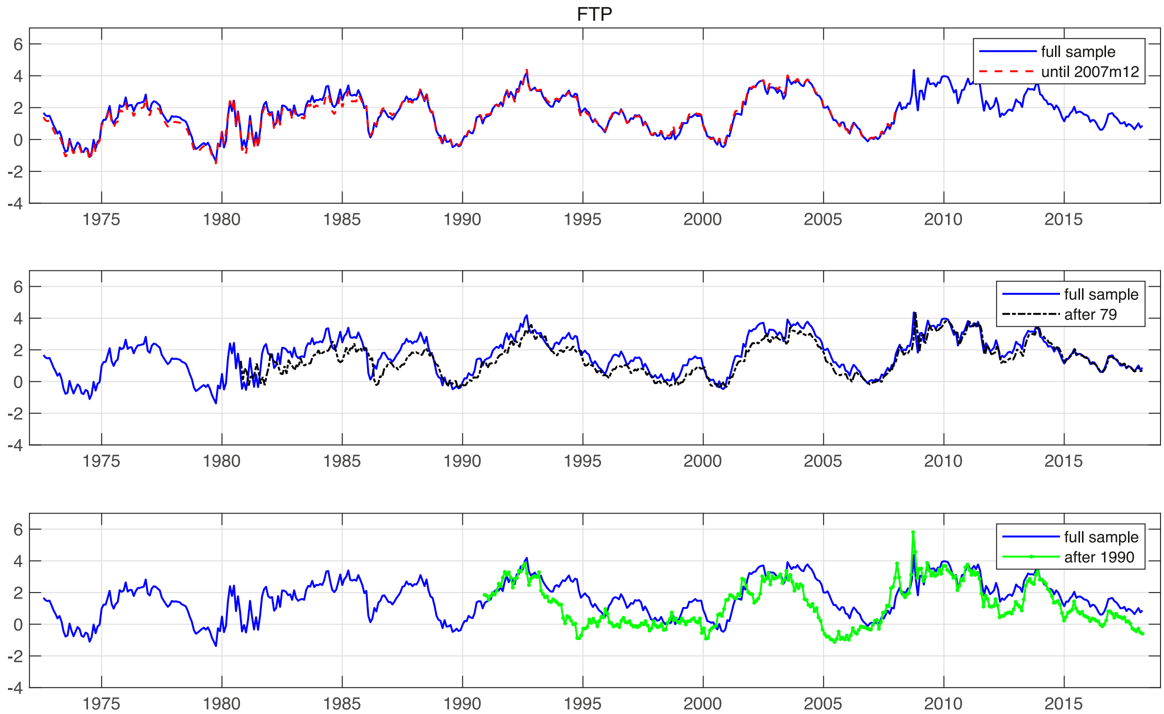

Fig. 8. Forward Term Premium Subsample Stability: FCVAR. This figure compares the baseline full sample term premium (10-5 forward term premium) with that associated with other subsamples: Pre-2008 (top graph), post-1979 (medium graph) and post-1990 (bottom graph).

on statistical grounds–. As a result, their long-term expectations of the short-rate are quite flat and most of the secular drop of the long-rate is attributed to the term premium. This is an important difference between our term premium and the other two which we also address throughout the paper (see also this point raised in Bauer et al., 2012, through a small-sample analysis of the I(0) VAR).

As a last exercise in this subsection, we derive the forward term premium for alternative subsamples. One potential limitation of both forward and yield term premium computations is that they may be quite sensitive to sample selection. We now elaborate on this point and compare our results with those of the two alternative forward term premia (I(1)-CVAR and I(0) VAR). Figure 8 shows the baseline forward term premia implied by our FCVAR model

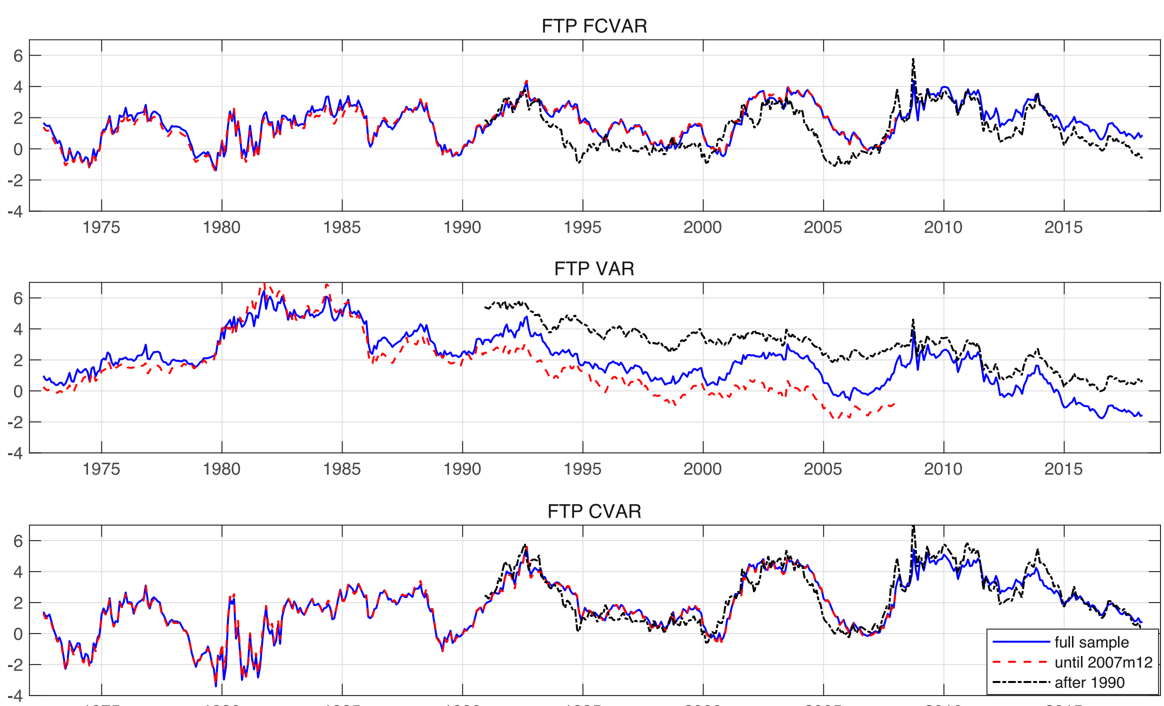

Fig. 9. Forward Term Premium Subsample Stability: FCVAR, VAR, CVAR. This figure compares the baseline full sample term premium (10-5 forward term premium) with that associated with other subsamples: FCVAR (top graph), I(0) VAR (medium graph) and I(1) CVAR (bottom graph). Each graph compares the full sample baseline forward term premium with the pre-2008 and the post-1990 counterparts.

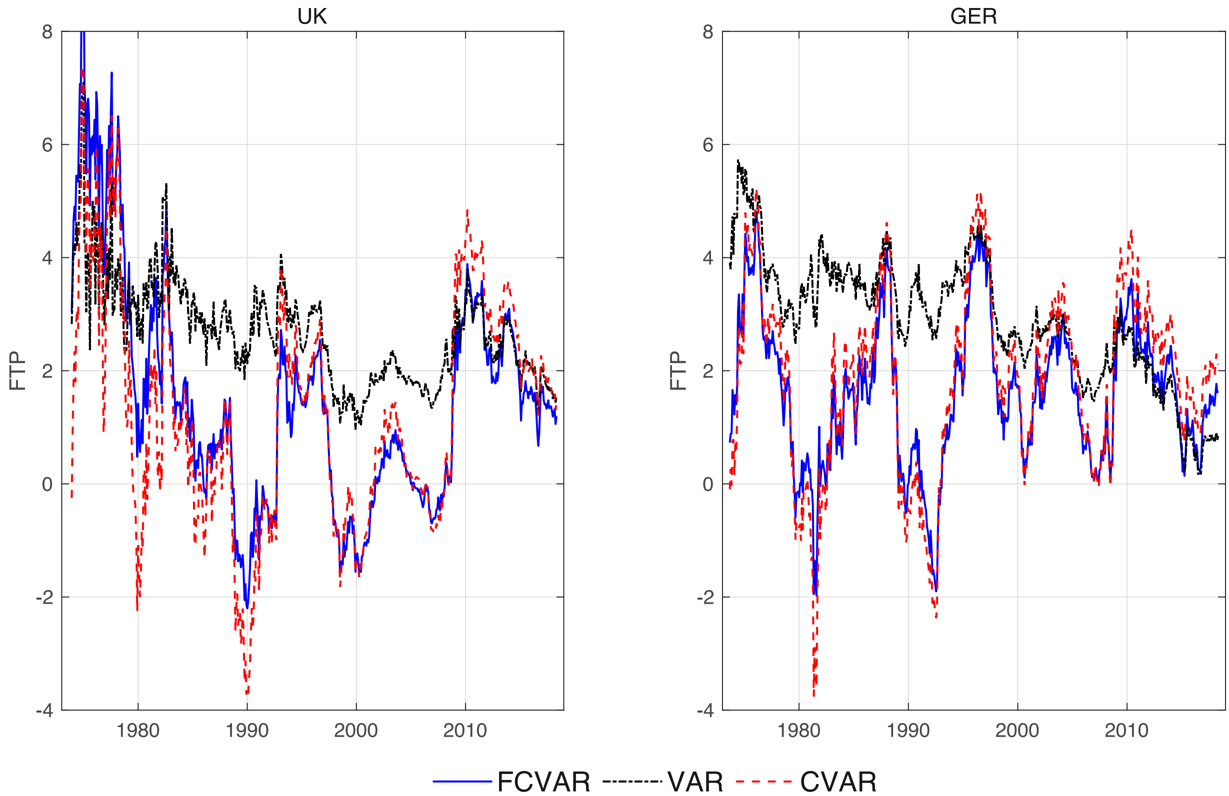

Fig. 10. Forward Term Premia: UK, Germany (FCVAR, I(0) VAR, I(1) CVAR). This figure plots the baseline implied term premia (10-5 forward term premia) across models (FCVAR, I(0) VAR, I(1) CVAR) for the UK (left graph) and Germany (right graph).

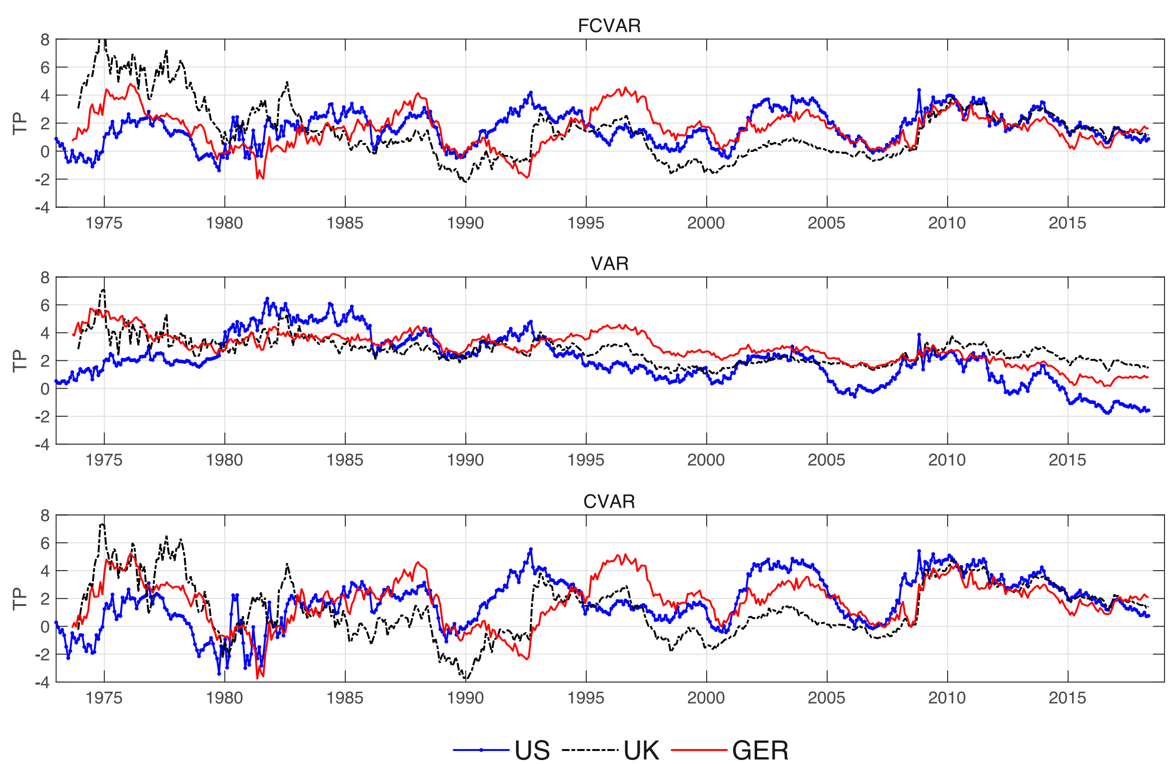

Fig. 11. Forward Term Premia: US, UK, Germany (FCVAR, I(0) VAR, I(1) CVAR). This figure plots the baseline (10-5 forward term premium) term premia of the three countries (US, UK, Germany) implied by the three models: FCVAR (top graph), I(0) VAR (medium graph) and CVAR (bottom graph).

Table 9
Diebold-Mariano Test: Recursive Scheme.

Accuracy of the out of sample forecast for the 1-year yield								
	Horizon in months							
	1	3	6	9	12	36	60	120
FCVAR	0.24	0.56	0.91	1.17	1.40	2.14	2.23	2.87
CVAR	0.28	0.67	1.02	1.25	1.45	2.11	2.07	2.45
VAR	0.31	0.72	1.10	1.45	1.85	3.68	4.63	5.28
DM - 1	4.85***	2.97***	1.57	1.62	2.05**	1.55	1.19	0.63
DM - 2	6.34***	3.38***	1.93*	1.08	0.60	-0.34	-1.62	-0.64
Accuracy of the out of sample forecast for the 3-year yield								
	Horizon in months							
	1	3	6	9	12	36	60	120
FCVAR	0.27	0.56	0.83	1.00	1.15	1.64	1.90	2.93
CVAR	0.29	0.63	0.88	1.04	1.17	1.56	1.68	2.50
VAR	0.31	0.68	0.98	1.29	1.63	3.29	4.44	5.26
DM - 1	4.19***	2.83***	1.62	2.14**	2.46**	1.60	1.17	0.63
DM -2	3.59***	2.56**	1.27	0.67	0.37	-0.91	-1.41	-0.64
Accuracy of the out of sample forecast for the 5-year yield								
	Horizon in months							
	1	3	6	9	12	36	60	120
FCVAR	0.25	0.51	0.72	0.84	0.96	1.30	1.72	2.83
CVAR	0.27	0.56	0.76	0.87	0.97	1.21	1.46	2.42
VAR	0.29	0.61	0.87	1.14	1.43	2.98	4.19	5.07
DM - 1	4.05***	2.80***	1.88*	2.41**	2.57**	1.62	1.17	0.63
DM - 2	2.92***	2.22**	1.04	0.57	0.25	-1.12	-1.27	-0.63
Accuracy of the out of sample forecast for the 10-year yield								
	Horizon in months							
	1	3	6	9	12	36	60	120
FCVAR	0.22	0.41	0.57	0.65	0.73	0.92	1.45	2.56
CVAR	0.23	0.45	0.61	0.69	0.75	0.84	1.21	2.21
VAR	0.25	0.49	0.72	0.95	1.15	2.51	3.69	4.62
DM - 1	3.62***	2.62***	2.27**	2.55**	2.54**	1.63	1.17	0.62
DM - 2	3.09***	2.12**	1.34	0.96	0.59	-1.73*	-1.20	-0.62

This table shows the Mean Absolute Errors (MAEs) of forecasts for the 1-year, 3-year, 5-year and 10-year Treasury yields and for different forecasting horizons, from h=1 month to h=120 months. The data are monthly from 1971:m8 to 2018:m4. The models are estimated with a recursive scheme, that is using an expanding estimation window using all data available up to each forecast date starting in 1986:m7 (when around 15 years of data are available) until 2018:m4. Last rows in each panel report the Diebold and Mariano (1995) statistic with a small-sample correction comparing the FCVAR with the VAR (DM-1) and the FCVAR with the CVAR (DM-2). Stars denote significance at the 1%(***), 5%(**) and 10%(*) level.

for alternative subsamples. The top graph compares the full-sample forward term premium with an alternative sample ending in 2008, right before the financial crisis. The two forward term premia turn out to be very similar across these two subsamples. The graphs below show the differences of forward term premia across two other subsamples (post-1979 and post-1990). In this latter subsample differences seem more noticeable, but overall dynamics are quite similar. We thus conclude that the FCVAR forward term premium is quite robust across subsamples despite relevant macro and monetary policy changes during our baseline sample.

Figure 9 shows the subsample forward term premia across models. It shows that the I(0) model –the most commonly used in forward term premium analysis– is the most unstable one, with some important discrepancies across subsamples. In contrast, the I(1) CVAR-implied term premia are very stable. This should come as no surprise since the I(1) model by construction depends almost entirely on the previous period interest rate, independently of the subsample taken.

5.3. Out-of-sample forecasting

How well does the FCVAR perform in out-of-sample forecasting of interest rates? We turn to this important question in this subsection, as we assess the performance of the FCVAR in predicting interest rates out-of-sample across maturities and forecast horizons relative to the I(0) VAR and the CVAR models. Relevant differences across models would have important implications for bond return, term premium and monetary policy predictability.

Table 9 shows the results of the Diebold and Mariano (1995) (DM) test of equal accuracy applied to our FCVAR, CVAR and VAR models. In order to derive the DM tests, the models are estimated with a recursive scheme, that is using an expanding estimation window using all data available up to each forecast date starting in 1986:m7 (when around 15 years of data are available) until 2018:m4. Then we report the Mean Absolute Errors (MAEs) of forecasts for the 1-year, 3-year, 5-year and 10-year Treasury yields and for different forecasting horizons, from h=1 month to h=120

Table 10
Diebold-Mariano Test: Rolling Scheme, Window =180 months.

Accuracy of the out of sample forecast for the 1-year yield								
	Horizon in months							
	1	3	6	9	12	36	60	120
FCVAR	0.23	0.51	0.81	1.00	1.20	2.22	2.46	3.16
CVAR	0.28	0.67	1.02	1.24	1.45	2.12	2.08	2.45
VAR	0.31	0.74	1.06	1.30	1.65	2.93	3.39	4.21
DM - 1	5.41***	3.91***	2.28**	2.00**	2.23**	1.05	0.90	0.59
DM - 2	5.44***	3.13***	2.08**	1.81*	1.57	-1.16	-1.38	-0.60
Accuracy of the out of sample forecast for the 3-year yield								
	Horizon in months							
	1	3	6	9	12	36	60	120
FCVAR	0.26	0.54	0.78	0.92	1.06	1.76	2.13	3.26
CVAR	0.29	0.63	0.89	1.04	1.17	1.56	1.68	2.50
VAR	0.32	0.71	0.97	1.20	1.50	2.65	3.23	4.17
DM - 1	4.60***	3.71***	2.18**	2.31**	2.51**	1.31	0.97	0.57
DM - 2	2.86***	2.17**	1.27	1.05	0.94	-1.27	-1.23	-0.60
Accuracy of the out of sample forecast for the 5-year yield								
	Horizon in months							
	1	3	6	9	12	36	60	120
FCVAR	0.26	0.50	0.71	0.82	0.92	1.46	1.98	3.15
CVAR	0.27	0.56	0.76	0.87	0.97	1.21	1.45	2.41
VAR	0.30	0.65	0.87	1.09	1.34	2.38	3.02	3.97
DM - 1	4.16***	3.53***	2.07**	2.33**	2.43**	1.37	0.95	0.55
DM - 2	1.55	1.65*	0.81	0.59	0.46	-1.34	-1.12	-0.59
Accuracy of the out of sample forecast for the 10-year yield								
	Horizon in months							
	1	3	6	9	12	36	60	120
FCVAR	0.22	0.42	0.60	0.69	0.77	1.13	1.73	2.89
CVAR	0.23	0.45	0.61	0.68	0.75	0.84	1.20	2.21
VAR	0.26	0.53	0.74	0.94	1.13	1.99	2.61	3.57
DM - 1	4.01***	3.28***	1.98**	2.21**	2.26**	1.35	0.90	0.51
DM - 2	1.11	1.07	0.19	-0.20	-0.30	-1.49	-1.08	-0.59

This table shows the Mean Absolute Errors (MAEs) of forecasts for the 1-year, 3-year, 5-year and 10-year Treasury yields and for different forecasting horizons, from h=1 month to h=120 months. The data are monthly from 1971:m8 to 2018:m4. The models are estimated with a rolling scheme with a window of 180 months. Last rows in each panel report the Diebold and Mariano (1995) statistic with a small-sample correction comparing the FCVAR with the VAR (DM-1) and the FCVAR with the CVAR (DM-2). Stars denote significance at the 1%(***), 5%(**) and 10%(*) level.

months. As in Bauer and Rudebusch (2020), we compute the DM test with a rectangular window for the long-run variance and the small-sample adjustment of Harvey et al. (1997). One problem is that for very long forecast horizons, the long-run variance is estimated with considerable uncertainty because there are only a few non-overlapping observations in our sample. This explains why the significance level of our results goes down for long horizons. Two key results emerge from Table 9. First, the FCVAR persistently beats the VAR in forecasting accuracy across interest rate maturities and forecast horizons (and statistically significant for $h \leq 12$ months). Second, the FCVAR beats the CVAR in forecasting accuracy for very short horizon, while the performances for $h \geq 6$ months are similarly accurate.

In the previous analysis, we adopted a recursive scheme, i.e. the size of the sample used in the estimation increases with each observation. To avoid the problem of competing forecasts using nested models, we repeated the same exercise using a rolling scheme, which uses only a fixed number of observations for parameter estimation. Specifically, we fixed the window to 180 observations (15 years). Table 10 shows the results of the analogous out-of-sample exercises. This new analysis confirms the previous results, highlighting the superiority of the FCVAR with respect to the I(0) VAR across maturities and at all horizons. The FCVAR also beats the I(1) CVAR at all maturities across most horizons below 1 year, while they perform similarly for longer horizons. All in all, the relative out-of-sample FCVAR performance is very good.

5.4. International term premia

The previous analysis suggests that the FCVAR model provides a realistic representation of the yield curve dynamics in the United States. An interesting question is whether these findings can be extended to other countries. To address this issue, in this section we estimate the FCVAR model for two other advanced economies, the UK and Germany. The data for the UK is from the Bank of England, and covers the sample from December 1972 to April 2018. The data for Germany is from the Bundesbank and covers the sample from September 1972 to April 2018. As for the US, results are based on a VAR(1) for short-run dynamics.

Table 11 shows the estimated values for the parameters d (estimation was also carried out under $d = b$). Our tests allow us to reject the hypothesis that interest rates are stationary I(0). While we do not report them for brevity, tests also reject the I(1) cointegration in favor of fractional cointegration.

Table 11
FCVAR Orders of Integration, US, UK and Germany.

	US	UK	GER
d	0.765 (0.050)	0.868 (0.036)	0.867 (0.037)

This table shows the estimates of the orders of integration of the FCVAR models for the three countries analyzed in this study: United States (US), United Kingdom (UK) and Germany (GER). Standard errors appear in parentheses.

We also test for the presence of fractional cointegration relationships between interest rates. As in the case of the US, we find the presence of a single fractional cointegrating relationship for the UK and Germany. The implied single cointegrating vectors for the UK and Germany are, respectively $\hat{\beta}'(UK) = [1, \quad -6.448, \quad 7.403, \quad -1.924]'$, and $\hat{\beta}'(GER) = [1, \quad -3.959, \quad 4.025, \quad -1.101]'$. Interestingly, the interpretation of the two estimated cointegrating vectors is essentially the same as in the US case. They imply that this model can accommodate both long-run yield curve parallel shifts and long-run slope shifts.

Figure 10 shows the risk neutral rates and forward term premia implied by the VAR, CVAR and FCVAR models for the UK and Germany. As was the case for the US, an accurate estimation of the persistence of interest rates is crucial for the identification and interpretation of forward term premium dynamics. For both countries, the I(0) forward term premium is relatively stable and presents a downward trend inherited by the interest rates. This happens because, by implying too fast mean-reversion of interest rate changes, the VAR model tends to underestimate the variability of risk neutral rates. On the contrary, the CVAR and FCVAR imply more volatile risk neutral rates and countercyclical forward term premia –probably too volatile in the case of the CVAR–. Importantly, the different models imply starkly different forward term premia dynamics. For example, in both countries forward term premia implied by the stationary VAR are around 4 percent higher than the ones implied by the FCVAR in the early 1980s and 1990s. The differences with the CVAR are smaller but still non-negligible.

As a final exercise, Fig. 11 shows the implied forward term premia of the three countries –the US, the UK and Germany– with the three models. Three facts stand out. First, in all cases the FCVAR model implies quite different forward term premia dynamics from the ones implied by the I(0) VAR. This is likely to be important for economic interpretation and policy-making. For example, in the US in 2018 the forward term premium is around -2 percent according to the VAR model while slightly positive according to the FCVAR model. Second, the US forward term premium is the least volatile of the three countries according to the FCVAR model, while the most volatile according to the VAR model. Finally, the FCVAR model implies a strong increase in synchronization and correlation starting with the financial crisis, while the forward term premia implied by the VAR model significantly depart after 2010.

6. Conclusions

This paper presents a yield curve model for interest rates capturing both a long run equilibrium relationship among the U.S. Treasury yields and the joint short-run dynamics. Our estimates of the flexible FCVAR model confirmed the existence of this pattern and characterized interest rates as a fractionally cointegrated and mean-reverting process. Our analysis also rejects some of the standard stationary I(0) and unit-root alternatives to joint modelling of interest rates. The estimates implied by our general FCVAR model are thus able to capture both the low-frequency movements in bond yields and the mean reversion commonly assumed in many financial models. We show that the implied forward term premium is quite robust to alternative subsamples and also derive the forward term premia for the UK and Germany.

As an important outcome of our exercise, this term structure model affords the identification of a credible term premium which can be readily used by both academics and policy makers. This can be done for different term premia maturities. As shown in the paper, term premia associated with longer maturities are non-linearly higher than shorter maturities. We also shed light on the sources of the term premium, which are mainly real, i.e. while economic growth lowers the term premium, economic slack and recessions increase the risk priced by investors in long-term bonds. Importantly, in terms of out-of-sample forecasting, the FCVAR beats the I(0) VAR model across interest rate maturities and horizons and the I(1) cointegrated VAR across maturities and short-horizons.

This article can be extended in several directions. Firstly, the term premium here has been specified in terms of a long memory property, characterized by a spectral density function which has a pole or singularity at the smallest (zero) frequency. However, it might be the case that the spectrum of the term premium contains peaks at other frequencies, referring, for example, to the business cycles. In this context, Gegenbauer (fractionally integrated) processes can be employed as an alternative to the standard I(d) approach used in this work. In addition, the FCVAR can be extended by adding other macro-finance variables potentially cointegrated with interest rates. Work in these directions is now in progress.

CRediT authorship contribution statement

Mirko Abbritti: Investigation, Data curation, Methodology, Formal analysis, Writing – review & editing. **Hector Carcel:** Investigation, Methodology, Data curation, Formal analysis. **Luis Gil-Alana:** Investigation, Methodology, Writing – review & editing. **Antonio Moreno:** Conceptualization, Investigation, Methodology, Writing – original draft.

Data availability

Data will be made available on request.

Appendix A. Fractional integration

Given a covariance stationary process $x_t, t = 0, \pm 1, \ldots$, a series has long memory if its spectral density function contains a pole or singularity at least at one frequency in the spectrum. Alternatively, it can be defined in the time domain by saying that x_t displays the property of long memory if the infinite sum of the autocovariances is infinite. A typical model satisfying the above two properties is the fractionally integrated or $I(d)$ model, where d is a positive value and can be formulated as:

$$(1-L)^d x_t = u_t, \quad t = 1, 2, \ldots, \tag{A.1}$$

with $x_t = 0$ for $t \leq 0$, where L represents the lag-operator, i.e. $L^k x_t = x_{t-k}$, and u_t is an $I(0)$ or short-memory process, defined in the frequency domain as a process with a spectral density function that is positive and bounded at all frequencies. Note that in this context, if $d > 0$, the spectral density function of x_t is unbounded at the smallest (zero) frequency, and the polynomial in the left hand side of equation (A.1) can be written for all real d as:

$$(1-L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = \left(1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 \ldots \right), \tag{A.2}$$

and thus:

$$(1-L)^d x_t = x_t - dx_{t-1} + \frac{d(d-1)}{2}x_{t-2} - \frac{d(d-1)(d-2)}{6}x_{t-3} + \ldots, \quad (A.3)$$

so that Eq. (A.1) can be expressed as:

$$x_t = dx_{t-1} - \frac{d(d-1)}{2}x_{t-2} + \frac{d(d-1)(d-2)}{6}x_{t-3} - \ldots + u_t. \quad (A.4)$$

Thus, the differencing parameter d plays a crucial role in describing the degree of dependence (persistence) in the data: The higher the value of d is, the higher the level of dependence between observations is. Three values of d are of particular interest. First, the case of $d = 0$ that implies short memory behaviour as opposed to the case of long memory with $d > 0$. Second, $d = 0.5$, since x_t becomes non-stationary as long as $d \geq 0.5$.[7] Finally, if $d < 1$ x_t is mean reverting with the effect of the shocks disappearing in the long-run, contrary to what happens if $d \geq 1$ with shocks having permanent effects and lasting forever.

Appendix B. A 4-Factor Model consistent with the FCVAR

As shown in Section 4, our results imply the existence of 3 stochastic trends and 1 cointegration relationship among the 4 yields in our estimation (the 1-year, 3-year, 5-year and 10-year yields). While there are alternative specifications consistent with the estimated FCVAR results, here we propose a particular time series model specification.

The model is constructed through 3 different underlying stochastic trends ($\tau_{1,t}, \tau_{2,t}$ and $\tau_{3,t}$) affecting yields. These stochastic trends have all the same order of integration d. In turn, yields are also affected by s_t, a stationary variable of order $d-b$. The first three model equations, for the 1, 3 and 10-year yields, become:

$$i_t^{(12)} = A_1 + B_{11}\tau_{1,t} + B_{12}s_t \quad (B.1)$$

$$i_t^{(36)} = A_2 + B_{21}\tau_{1,t} + B_{22}\tau_{2,t} + B_{23}s_t \quad (B.2)$$

$$i_t^{(120)} = A_3 + B_{31}\tau_{1,t} + B_{32}\tau_{3,t} + B_{33}s_t. \quad (B.3)$$

This set of three equations imply three independent stochastic trends for the three yields, each of which is an $I(d)$ process. Since each of these three yields depends on different underlying stochastic trends, they are not cointegrated among themselves. However, and following our estimates, a linear combination of these three yields and the 5-year yield ($i_t^{(60)}$) is cointegrated, so that:

$$\beta' Y_t = s_t, \quad (B.4)$$

where $Y_t = [i_t^{(12)} \quad i_t^{(36)} \quad i_t^{(60)} \quad i_t^{(120)}]'$, and given that $\hat{\beta}' = [1, \quad -2.598, \quad 2.347, \quad -0.760]'$,

$$i_t^{(60)} = a + b_1\tau_{1,t} + b_2\tau_{2,t} + b_3\tau_{3,t} + cz_t \quad (B.5)$$

where:

$$a = \frac{2.598A_2 + 0.76A_3 - A_1}{2.347}$$

$$b_1 = \frac{2.598B_{21} + 0.76B_{31} - B_{11}}{2.347}$$

$$b_2 = \frac{2.598B_{22}}{2.347}$$

$$b_3 = \frac{0.76B_{32}}{2.347}$$

and $c = \dfrac{2.598B_{23} + 0.76B_{33} - B_{12}}{2.347}$.

This 4-factor model is consistent with the underlying model of Johansen (2008) (Section 1), which gives rise to the FCVAR which we estimated. Notice that in this particular model, there is a stochastic trend $\tau_{1,t}$ affecting all yields. This stochastic trend could be seen as the traditional level factor, which in the recent work of Bauer and Rudebusch (2020) is an I(1) variable shifting all yields in similar amounts. Additionally, our model displays two other underlying stochastic trends ($\tau_{2,t}$ and $\tau_{3,t}$), which enrich the model dynamics. These two trends affect the 3, 5 and 10 year yields and can also have a long-run effect in the yield curve. Only the 5-year yield includes all stochastic trends, capturing all stochastic trends present in both short and long-term yields.[8]

Finally, the yield curve slope loads on the three stochastic trends: The 3-year slope loads on $\tau_{1,t}$ and $\tau_{2,t}$, the 10-year slope loads on $\tau_{1,t}$ and $\tau_{3,t}$, whereas the 5-year slope loads on $\tau_{1,t}$, $\tau_{2,t}$ and $\tau_{3,t}$. Under similar loadings of yields on $\tau_{1,t}$, long-run yield curve shifts can accommodate approximately parallel shifts in the yield curve. For different loadings across maturities, this model can accommodate persistent changes in the slope dynamics. To sum up, this model can accommodate both permanent changes in the level and the slope of the term structure of interest rates.

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[7] It is non-stationary in the sense that the variance of the partial sums increases in magnitude with d.

[8] Our specific allocation of the stochastic trends τ_2 and τ_3 is motivated by the proximity of the yields. So τ_2 affects the 3 and 5-year yields (short and medium-term) whereas τ_3 affects the 5 and 10-year yields (medium and long-term).

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