



The effective cross section for double parton scattering within a holographic AdS/QCD approach



Marco Traini ^{a,b,*}, Matteo Rinaldi ^c, Sergio Scopetta ^{d,e}, Vicente Vento ^c

^a Institut de Physique Théorique, Université Paris Saclay, CEA, F-91191 Gif-sur-Yvette, France

^b INFN - TIFPA, Dipartimento di Fisica, Università degli Studi di Trento, Via Sommarive 14, I-38123 Povo, Trento, Italy

^c Departament de Física Teòrica, Universitat de València and Institut de Física Corpuscular, Consejo Superior de Investigaciones Científicas, 46100 Burjassot, València, Spain

^d Dipartimento di Fisica e Geologia, Università degli Studi di Perugia, Via A. Pascoli, I-06123, Italy

^e Istituto Nazionale di Fisica Nucleare, Sezione di Perugia, Perugia, Italy

ARTICLE INFO

Article history:

Received 14 October 2016

Received in revised form 16 February 2017

Accepted 25 February 2017

Available online 3 March 2017

Editor: J.-P. Blaizot

ABSTRACT

A first attempt to apply the AdS/QCD framework for a bottom-up approach to the evaluation of the effective cross section for double parton scattering in proton–proton collisions is presented. The main goal is the analytic evaluation of the dependence of the effective cross section on the longitudinal momenta of the involved partons, obtained within the holographic Soft-Wall model. If measured in high-energy processes at hadron colliders, this momentum dependence could open a new window on 2-parton correlations in a proton.

© 2017 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP³.

1. Introduction

The effects of multiple parton interactions (MPI) in proton–proton scattering have been the object of several studies which have a long history (see, e.g. Ref. [1]) and, at the same time, continue to be an active field of interest. From an experimental point of view, the Large Hadron Collider (LHC) has opened the possibility to observe specific signatures of these effects (see [2–6] for recent reports), useful to constrain the background for the search of New Physics; from a theoretical point of view, the investigation of two-parton correlations will become possible, opening a new field in the description of the non-perturbative three dimensional (3D) proton structure (see, e.g., Ref. [7]). The simplest MPI process is double parton scattering (DPS), whose description is based on specific non-perturbative elements: the double Parton Distribution Functions (dPDFs). These quantities describe the number densities of two partons, located at a given transverse distance (b_{\perp}) in coordinate space, which carry given longitudinal momentum fractions ($x_i = x_1, x_2$) of the parent proton. The calculation of dPDFs, non-perturbative quantities, is particularly cumbersome and therefore one can perform model calculations able to focus on the relevant features [8–11]. Usually, in the literature, the Fourier transform of the dPDFs w.r.t. b_{\perp} , depending therefore on k_{\perp} , the relative

transverse momentum between the two acting partons, sometimes called ${}_2$ GPDs, has been studied. At present, it has not yet been possible to extract dPDFs from experimental data, but a specific observable, related to DPS, has received much attention in the past: the so called effective cross section, σ_{eff} . It is defined through the ratio of the product of two single parton scattering cross sections to the DPS cross section with the same final states and can be parameterized in terms of dPDFs and parton distribution functions (PDFs). The effective cross section has been extracted, although in a model dependent way, in several experiments [12–17]. The apparent conclusion, within the present scenario and despite the large error bars, is that σ_{eff} remains constant as a function of the center-of-mass energy of the collision.

In Ref. [18] we have recently investigated σ_{eff} , using the dPDFs calculated within the Light-Front (LF) approach developed in Ref. [10]. A clear dependence on the fractions of proton longitudinal momentum carried by the four partons involved in the DPS process has been predicted. This feature could represent a first access to the experimental observation of two-parton correlations in the proton.

The aim of the present work is to provide confirmation on the x_i dependence in σ_{eff} by using an AdS/QCD framework, a completely different approach to hadron structure than the LF formalism used in Ref. [18]. The leitmotif of the AdS/QCD approach is the duality between conformal field theories and gravitation in an anti de Sitter space [19]. Since QCD is not a conformal theory, peo-

* Corresponding author.

E-mail address: marcoclaudio.traini@unitn.it (M. Traini).

ple have not been able to develop yet the fundamental top–down approach. We shall proceed therefore by a bottom–up approach where important features of QCD are implemented generating a theory in which conformal symmetry is only asymptotically restored [20,21]. In this scheme we make use of the well established Soft-Wall model [22] of the AdS/CFT framework. Within this approach it has been proven that the gauge/gravity duality provides a (holographic) mapping of the string model $\Phi(z)$, z being the fifth dimension, to the hadron Light-Front wave functions (LFWFs) in four dimensional space-time. The approach has been successfully applied to the description of the mass spectrum of mesons and baryons reproducing the Regge trajectories (e.g. Ref. [21] and references therein) and to Deep Inelastic scattering for the evaluation of the Generalized Parton Distributions (GPDs) (e.g. Refs. [23–27]). This last result can be used to link AdS/QCD and double parton physics as described in the next section 2 where we propose to calculate dPDFs within a general factorization framework which makes use of the GPDs as calculated in the AdS/QCD holographic approach. In section 3 we investigate explicitly the effective cross section calculating its x_i -dependence in a simple and analytic way and, eventually, conclusions are drawn in section 4.

2. dPDFs from GPDs in AdS/QCD

The approach we are using, a semiclassical approximation to QCD, is often called Light Front Holography (LFH) [28]. It is based on the realization of a mapping relating AdS modes to LFWFs; it is obtained by matching specific matrix elements (e.g. the electromagnetic form factors) in the two approaches – string theory in AdS and Light-Front QCD in Minkowski space-time [29]. An interesting application of the gauge/gravity correspondence to hadronic properties in the strong coupling regime, where QCD cannot be used in a direct and simple way, is the calculation of the Generalized Parton Distributions (GPDs) of the nucleon, described in Ref. [23–27] and we refer to those references for the detailed aspects of the calculation of GPDs and of the holographic mapping. In the next sections, we will recall only basic results to be used in the study of dPDFs.

2.1. GPDs in SW-model

Expressions for the GPDs in terms of the AdS modes are obtained making use of the holographic mapping suggested by Brodsky and de Teramond [29] for the Hadron electromagnetic form factors (see also Ref. [30] for recent developments). The calculation of the nucleon form factors is, in fact, based on the use of the integral representation for the bulk-to-boundary propagator introduced by Grigoryan and Radyushkin [31]:

$$V(k_{\perp}^2, \zeta) = \int_0^1 dx F_x(k_{\perp}^2, \zeta) = \int_0^1 dx \frac{(\alpha\zeta)^2}{(1-x)^2} x^{k_{\perp}^2/(4\alpha^2)} e^{-(\alpha\zeta)^2 x/(1-x)}, \quad (1)$$

where α is the parameter that appears in the dilaton definition used to break conformal invariance in AdS. α affects all fields considered in the model, including the vector massless field which allows the calculation of form factors (and GPDs). The same parameter appears, in the case of the nucleon, in the Soft-Wall potential, in the holographic coordinate $V_{SW}(z) = \alpha^2 z$. We fix its numerical value according to Refs. [25,27], $\alpha = 0.41$ GeV.

GPDs parameterize the non-perturbative hadron structure in hard exclusive processes [32,33]. We recall that GPDs depend on

the longitudinal momentum fraction of the active quark, x , on the momentum transferred in the longitudinal direction (ξ , the so called skewness), on the invariant momentum transfer, t , and on the momentum scale μ_0^2 . In the following, the latter dependence will be omitted for simplicity, if not differently specified.

The first t -dependent moments of GPDs are related to the nucleon elastic form factors, i.e.

$$\int_{-1}^1 dx H^q(x, \xi, t) = F_1^q(t), \quad \int_{-1}^1 dx E^q(x, \xi, t) = F_2^q(t), \quad (2)$$

where $F_1^q(t)$ and $F_2^q(t)$ are the contributions of quark q to the Dirac and Pauli form factors, respectively. The property Eq. (2) does not depend on ξ and it holds also at $\xi = 0$. Introducing the so-called valence GPDs, $H_V^q(x, \xi, t) = H^q(x, \xi, t) + H^q(-x, \xi, t)$ (analogously one can define $E_V^q(x, \xi, t)$), whose forward limit is $H_V^q(x, \xi = 0, t = 0) = q_V(x) = q(x) - \bar{q}(x)$, and assuming isospin symmetry, from Eq. (2) one gets

$$\begin{aligned} F_1^p(t) &= \int_0^1 dx \left(+\frac{2}{3} H_V^u(x, \xi=0, t) - \frac{1}{3} H_V^d(x, \xi=0, t) \right), \\ F_1^n(t) &= \int_0^1 dx \left(-\frac{1}{3} H_V^u(x, \xi=0, t) + \frac{2}{3} H_V^d(x, \xi=0, t) \right), \\ F_2^p(t) &= \int_0^1 dx \left(+\frac{2}{3} E_V^u(x, \xi=0, t) - \frac{1}{3} E_V^d(x, \xi=0, t) \right), \\ F_2^n(t) &= \int_0^1 dx \left(-\frac{1}{3} E_V^u(x, \xi=0, t) + \frac{2}{3} E_V^d(x, \xi=0, t) \right). \end{aligned} \quad (3)$$

Eqs. (3) and (2), allow for the extraction of the functions $H_V^{u,d}(x, \xi=0, t, \mu_0^2)$ and $E_V^{u,d}(x, \xi=0, t, \mu_0^2)$ at the scale μ_0^2 .

As a conclusion, the helicity independent GPDs H_V^q assume the explicit form [23,25,27]:

$$\begin{aligned} H_V^u(x, \xi = 0, t, \mu_0^2) &= u_V(x, \mu_0^2) x^{-\frac{t}{4\alpha^2}}, \\ H_V^d(x, \xi = 0, t, \mu_0^2) &= d_V(x, \mu_0^2) x^{-\frac{t}{4\alpha^2}}. \end{aligned} \quad (4)$$

Analogous expressions can be written for the target helicity-flip GPDs E_V^q .

One should notice that, in the obtained GPDs, the dependence on the longitudinal momentum and on the momentum transfer are not factorized, as it happens, to our knowledge, in all the microscopic model calculations of GPDs (see, e.g., Refs. [34] and [35]).

2.2. Factorization

As already mentioned, in actual analyses, dPDFs are usually approximated by factorized forms. In particular, as firstly proposed in Ref. [36] and widely used, the dPDF in momentum space, $F_{u_V u_V}(x_1, x_2, k_{\perp}, \mu_0^2)$, can be written as a product of two spin independent, quark helicity conserving GPDs $H_V^u(x, \xi = 0, k_{\perp}, \mu_0^2)$:

$$\begin{aligned} F_{u_V u_V}(x_1, x_2, k_{\perp}, \mu_0^2) &= \\ \approx H_V^u(x_1, \xi = 0, -k_{\perp}^2, \mu_0^2) H_V^u(x_2, \xi = 0, -k_{\perp}^2, \mu_0^2). \end{aligned} \quad (5)$$

As indicated, GPDs depend also on the momentum scale μ_0 .¹ To be more precise, let us concentrate first on the chiral even (helicity conserving) distribution $H_V^q(x, \xi, t, Q^2)$ for partons of q -flavor, and taking deeply virtual Compton scattering (DVCS) as a typical process. A virtual photon of momentum q_μ is exchanged by a lepton to a nucleon of momentum P_μ and a real photon of momentum q'_μ is produced, together with a recoiling nucleon with momentum P'_μ . The space-like virtuality is therefore $Q^2 = -q_\mu q^\mu$ and it identifies the scale of the process (in the expression (5), $Q^2 = \mu_0^2$). The invariant momentum transfer is $t = -k_\perp^2 = (P'_\mu - P_\mu)^2$ and the skewedness ξ encodes the change of the longitudinal nucleon momentum ($2\xi = k^+/\bar{P}^+$, with $2\bar{P}_\mu = (P_\mu + P'_\mu)$).

The factorized form (5) contains only the GPDs at $\xi = 0$; it is remarkable that, when Fourier transformed to coordinate space, these quantities become densities, the so called impact parameter dependent parton distributions (the reader can find in Ref. [33] a recent update on GPDs physics). It is also interesting to note that the dPDF, Eq. (5), Fourier transformed to coordinate space, is given by a convolution of impact parameter dependent parton distributions. In this approximation, the longitudinal momenta of the quarks described by the dPDF are not correlated, while these momenta and \mathbf{k}_\perp are correlated (see Ref. [3] for a discussion on this issue).

The H_V^u are normalized in the natural way

$$\int dx H_V^u(x, \xi = 0, k_\perp^2 = 0, \mu_0^2) = 2,$$

$$\int dx H_V^d(x, \xi = 0, k_\perp^2 = 0, \mu_0^2) = 1,$$

and the factorization (5) is valid in the region $x_1 + x_2 < 1$, i.e. in the region kinematically accessible to the two partons whose total momentum cannot exceed the nucleon momentum.

In Ref. [3] also a first order correction to Eq. (5) has been evaluated and the total expression reads

$$F_{uvuv}(x_1, x_2, k_\perp, \mu_0^2) \approx H_V^u(x_1, \xi = 0, -k_\perp^2, \mu_0^2) H_V^u(x_2, \xi = 0, -k_\perp^2, \mu_0^2) + \frac{k_\perp^2}{4M_p^2} E_V^u(x_1, \xi = 0, -k_\perp^2, \mu_0^2) E_V^u(x_2, \xi = 0, -k_\perp^2, \mu_0^2), \quad (6)$$

which includes a correction containing E_V^q , the nucleon spin independent, target helicity flip GPD, and M_p is the proton mass.

3. x_i -Dependence of the proton effective cross section

The effective cross section, σ_{eff} , is a relevant quantity in the experimental analysis of DPS (for a recent update, see, e.g., Ref. [18] and references therein).

An expression for σ_{eff} , suitable for theoretical evaluations, has been developed in Ref. [18] and can be written as follows:

$$\sigma_{eff}(x_1, x'_1, x_2, x'_2, \mu_0^2) = \frac{\sum_{i,k,j,l} C_{ik} C_{jl} F_i(x_1) F_k(x'_1) F_j(x_2) F_l(x'_2)}{\sum_{i,j,k,l} C_{ik} C_{jl} \int F_{ij}(x_1, x_2, k_\perp) F_{kl}(x'_1, x'_2, -k_\perp) \frac{d\mathbf{k}_\perp}{(2\pi)^2}}, \quad (7)$$

where F_i, F_k, F_j, F_l are the PDFs entering the process in study (globally, $i, k, j, l = q, \bar{q}, g$), $F_{ij}(x_1, x_2, k_\perp)$ are the related dPDFs (in

¹ In principle, dPDFs depend on two momentum scales, corresponding to those of the two different processes which are produced by the two active partons in the DPS process. Nevertheless, we assume here for definiteness that the two scales coincide.

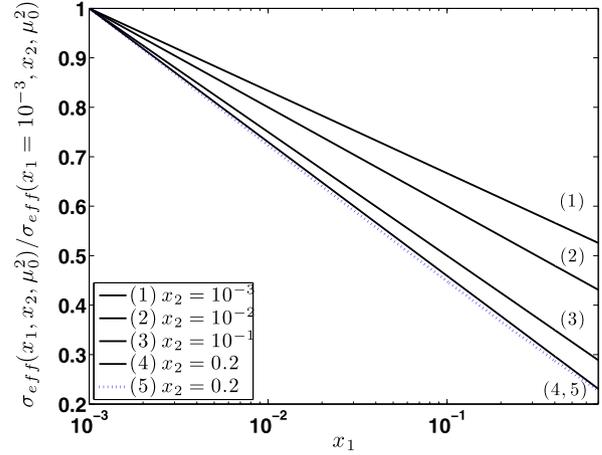


Fig. 1. (Color online) $\sigma_{eff}(x_1, x_2, \mu_0^2)$ (normalized at $x_1 = x_2 = 10^{-3}$, Eq. (9)), as a function of x_1 at fixed $x_2 = 0.001, 0.01, 0.1, 0.2$. The small contribution due to the higher order term in Eq. (10) is shown for $x_2 = 0.2$ (dotted). The low- x region is emphasized by means of a logarithmic x_1 -scale.

Eq. (7) the explicit dependence on the scale μ_0^2 has been suppressed for simplicity) and C_{ik} are color factors. In principle, σ_{eff} depends on four momentum fractions. In order to discuss the main features of σ_{eff} , one can restrict the analysis to the zero rapidity region ($y = 0$), and therefore to $x_i = x'_i$, and to valence uv which remains the dominant component of the Fock space in the AdS approach and it is identified with valence quarks [27,37]:

$$\begin{aligned} \sigma_{eff}(x_1, x_2, \mu_0^2) &\simeq \\ &\simeq \frac{[u_V(x_1, \mu_0^2)u_V(x_2, \mu_0^2)]^2}{\int [F_{uvuv}(x_1, x_2, k_\perp, \mu_0^2)]^2 \frac{d\mathbf{k}_\perp}{(2\pi)^2}} = \\ &= \frac{1}{\int x_1^{k_\perp^2/(2\alpha^2)} x_2^{k_\perp^2/(2\alpha^2)} \frac{d\mathbf{k}_\perp}{(2\pi)^2}}, \end{aligned} \quad (8)$$

where the explicit dependence (4) has been used. Eq. (8) shows that an analytic dependence on x_1, x_2 is predicted by the holographic AdS approach.

In particular in the valence region, the behavior is qualitatively similar to the one found previously within a LF approach [18]. Quantitatively, taking for example $x_1 = x_2 = 0.4$, one finds, from (8) $\sigma_{eff} \simeq \frac{2\pi}{\alpha^2} [\ln(1/x_1) + \ln(1/x_2)] \simeq 26.6$ mbarn, a value which is not far from the result of Ref. [18] and from those extracted by the experimental collaborations. As shown in Ref. [18], at least in the valence region, QCD evolution does not change substantially the x -dependence and the absolute values of σ_{eff} .

However, in the present analysis, we are especially interested in the x_i dependence of σ_{eff} , which is found to be a largely model independent feature. To that aim we normalize the cross section at some low- x_2 value ($x_2 = x_2^0$) obtaining (for $x_1 + x_2 < 1$, and $x_1 \geq 10^{-3}$)

$$\frac{\sigma_{eff}(x_1 \geq 10^{-3}, x_2 = x_2^0, \mu_0^2)}{\sigma_{eff}(x_1 = x_1^0, x_2 = x_2^0, \mu_0^2)} = \frac{\ln(1/x_1) + \ln(1/x_2)}{\ln(10^3) + \ln(1/x_2)}, \quad (9)$$

which represents the essential result of the present work, illustrated in Fig. 1, where the ratios (9), is shown as a function of x_1 and at different values of x_2 . A relevant x_i -dependence of the cross section is found. It turns out to be rather strong in the valence region, as already indicated in Ref. [18]. It is important to notice that this dependence is sizable also at lower values of x_i , manifesting a suppression of 20–30% at $x_1 = 0.01$ (depending on the value of x_2). At $x_1 = 0.1$, the suppression is around 50%.

Before concluding the section let us discuss the further correction due to the k_{\perp}^2/M_p^2 contribution in Eq. (6), as proposed in Ref. [3]. An explicit calculation shows that Eq. (9) still holds with the simple replacement

$$\ln(1/x_1) + \ln(1/x_2) \rightarrow \frac{\ln(1/x_1) + \ln(1/x_2)}{1 + \left(\frac{\alpha}{2M_p}\right)^2 \frac{f(x_1, x_2)}{\ln(1/x_1) + \ln(1/x_2)}}, \quad (10)$$

where

$$f(x_1, x_2) = (3\kappa_u)^2 \frac{(1-x_1)^2(1-x_2)^2}{(1-x_1)(13/3-x_1)(1-x_2)(13/3-x_2)}, \quad (11)$$

and $\kappa_u = 2\kappa_p + \kappa_n \approx 1.673$ is related to the anomalous magnetic moments of proton and neutron, κ_p and κ_n , respectively.

The correction is very small as it can be seen in Fig. 1, where its effects are shown for $x_2 = 0.2$ (the other cases being quite similar).

4. Summary and conclusions

The present work addresses a topic which has a specific relevance in extracting double parton correlations from high-energy proton–proton scattering data: the x_i -dependence of the (so called) effective cross section, a dependence put in numerical evidence in Ref. [18]. The relevance of such a dependence deserves some further study and we have investigated it within an AdS/QCD holographic approach. In fact it is largely recognized that such a technique is a good analytic tool to investigate physical systems, and their electromagnetic interactions, within non-perturbative QCD (see Ref. [21] for a recent report). The approach here proposed applies, for the first time, AdS/QCD to the evaluation of dPDFs and parton correlations. The result is rather direct, showing a clear x_i dependence of the effective cross section. Experimentally, such a dependence is not evident, most likely because of the large error bars. A better identification of the behavior of the cross section as a function of the center-of-mass energy of the collision would open interesting windows on the parton–parton correlations and, consequently, on a novel way to look at specific features of the 3-D structure of the nucleon.

Acknowledgements

M.T. thanks the members of the IPhT-C.E.A. Saclay for the friendly hospitality during a visiting period in the summer 2016, when this work has been done. This work was supported in part by the MINECO under contract FPA2013–47443–C2–1–P, by GVA-PROMETEOII/2014/066 and SEV-2014–0398. S.S. thanks the Department of Theoretical Physics of the University of Valencia for warm hospitality and support. M.T. and V.V. thank the INFN, sezione di Perugia and the Department of Physics and Geology of the University of Perugia for warm hospitality and support. Several discussions with F.A. Ceccopieri are gratefully acknowledged.

References

- [1] N. Paver, D. Treleani, *Nuovo Cimento A* 70 (1982) 215.
- [2] J.R. Gaunt, W.J. Stirling, *J. High Energy Phys.* 03 (2010) 005.

- [3] M. Diehl, D. Ostermeier, A. Schafer, *J. High Energy Phys.* 03 (2012) 089.
- [4] A.V. Manohar, W.J. Waalewijn, *Phys. Rev. D* 85 (2012) 114009.
- [5] S. Bansal, P. Bartalini, B. Blok, D. Ciangottini, M. Diehl, F.M. Fionda, J.R. Gaunt, P. Gunnellini, et al., arXiv:1410.6664 [hep-ph].
- [6] A. Szczurek, arXiv:1504.06491 [hep-ph].
- [7] G. Galucci, D. Treleani, *Phys. Rev. D* 60 (1999) 054023.
- [8] H.M. Chang, A.V. Manohar, W.J. Waalewijn, *Phys. Rev. D* 87 (2013) 034009.
- [9] M. Rinaldi, S. Scopetta, V. Vento, *Phys. Rev. D* 87 (2013) 114021.
- [10] M. Rinaldi, S. Scopetta, M. Traini, V. Vento, *J. High Energy Phys.* 12 (2014) 028; M. Rinaldi, S. Scopetta, M. Traini, V. Vento, *J. High Energy Phys.* 10 (2016) 063; M. Rinaldi, F.A. Ceccopieri, *Phys. Rev. D* (2017), in press, arXiv:1611.04793 [hep-ph].
- [11] W. Broniowski, E. Ruiz Arriola, *Few-Body Syst.* 55 (2014) 381; W. Broniowski, E. Ruiz Arriola, K. Golec-Biernat, *Few-Body Syst.* 57 (2016) 405.
- [12] T. Akesson, et al., Axial Field Spectrometer Collaboration, *Z. Phys. C* 34 (1987) 163.
- [13] J. Alitti, et al., UA2 Collaboration, *Phys. Lett. B* 268 (1991) 145.
- [14] F. Abe, et al., CDF Collaboration, *Phys. Rev. D* 56 (1997) 3811.
- [15] V.M. Abazov, et al., D0 Collaboration, *Phys. Rev. D* 81 (2010) 052012.
- [16] G. Aad, et al., ATLAS Collaboration, *New J. Phys.* 15 (2013) 033038.
- [17] S. Chatrchyan, et al., CMS Collaboration, *J. High Energy Phys.* 03 (2014) 032.
- [18] M. Rinaldi, S. Scopetta, M. Traini, V. Vento, *Phys. Lett. B* 752 (2016) 40.
- [19] J.M. Maldacena, *Adv. Theor. Math. Phys.* 2 (1998) 231, arXiv:hep-th/9711200; S.S. Gubser, I.R. Klebanov, A.M. Polyakov, *Phys. Lett. B* 428 (1998) 105, arXiv:hep-th/9802109; E. Witten, *Adv. Theor. Math. Phys.* 2 (1998) 253, arXiv:hep-th/9802150, *Adv. Theor. Math. Phys.* 2 (1998) 505, arXiv:hep-th/9803131.
- [20] U. Gursoy, E. Kiritsis, *J. High Energy Phys.* 0802 (2008) 032, arXiv:hep-th/0707132; U. Gursoy, E. Kiritsis, F. Nitti, *J. High Energy Phys.* 0802 (2008) 019, arXiv:hep-th/0707134.
- [21] S.J. Brodsky, G.F. de Teramond, H.G. Dosch, J. Erlich, *Phys. Rep.* 584 (2015) 1, arXiv:hep-ph/1407813.
- [22] J. Erlich, E. Katz, D.T. Son, M.A. Stephanov, *Phys. Rev. Lett.* 95 (2005) 261602, arXiv:hep-ph/0501128; L. Da Rold, A. Pomarol, *Nucl. Phys. B* 721 (2005) 79, arXiv:hep-ph/0501218; A. Karch, E. Katz, D.T. Son, M.A. Stephanov, *Phys. Rev. D* 74 (2006) 015005, arXiv:hep-ph/0602229.
- [23] A. Vega, I. Schmidt, T. Gutsche, V.E. Lyubovitskij, *Phys. Rev. D* 83 (2011) 036001, arXiv:hep-ph/1010281.
- [24] A. Vega, I. Schmidt, T. Gutsche, V.E. Lyubovitskij, *Phys. Rev. D* 85 (2012) 096004, arXiv:hep-ph/1202480, *Phys. Rev. D* 88 (2013) 073006, arXiv:hep-ph/1307512; R. Nishio, T. Watari, *Phys. Lett. B* 707 (2012) 362, arXiv:hep-ph/1105290.
- [25] D. Chakrabarti, C. Mondal, *Phys. Rev. D* 88 (2013) 073006, arXiv:hep-ph/1307512.
- [26] Neetika Sharma, *Phys. Rev. D* 90 (2014) 095024, arXiv:hep-ph/1411748.
- [27] M. Traini, arXiv:1608.08410 [hep-ph].
- [28] S.J. Brodsky, G.F. de Teramond, *Phys. Rev. Lett.* 96 (2006) 201601, arXiv:hep-ph/0602252; S.J. Brodsky, G.F. de Teramond, *Phys. Lett. B* 582 (2004) 211, arXiv:hep-th/0310227.
- [29] S.J. Brodsky, G.F. de Teramond, *Phys. Rev. D* 77 (2008) 056007, arXiv:hep-ph/0707385.
- [30] R.S. Sufian, G.F. de Teramond, S.J. Brodsky, A. Deur, H.G. Dosch, arXiv:hep-ph/1609066.
- [31] H.R. Grigoryan, A.V. Radyushkin, *Phys. Rev. D* 76 (2007) 095007, arXiv:hep-ph/0706154.
- [32] M. Guidal, M.V. Polyakov, A.V. Radyushkin, M. Vanderhaeghen, *Phys. Rev. D* 72 (2005) 054013, arXiv:hep-ph/0410251.
- [33] M. Diehl, *Eur. Phys. J. A* 52 (6) (2016) 149, arXiv:1512.01328 [hep-ph].
- [34] S. Scopetta, V. Vento, *Eur. Phys. J. A* 16 (2003) 527, arXiv:hep-ph/0201265.
- [35] S. Boffi, B. Pasquini, M. Traini, *Nucl. Phys. B* 649 (2003) 243, arXiv:hep-ph/0207340.
- [36] B. Blok, Y. Dokshitzer, L. Frankfurt, M. Strikman, *Phys. Rev. D* 83 (2011) 071501, arXiv:1009.2714 [hep-ph].
- [37] T. Gutsche, V.E. Lyubovitskij, I. Schmidt, A. Vega, *Phys. Rev. D* 86 (2012) 036007, arXiv:hep-ph/1204661.