The published version of the paper "García-Marín A. P., Morbidelli R., Saltalippi C., Cifrodelli M., Estévez J., Flammini A. (2019). On the choice of the optimal frequency analysis of annual extreme rainfall by multifractal approach. Journal of Hydrology, 575, 1267-1279." is available at: https://doi.org/10.1016/j.jhydrol.2019.06.013

On the choice of the optimal frequency analysis of annual extreme rainfall by multifractal approach

Amanda P. García-Marín[1,2*], Renato Morbidelli[2], Carla Saltalippi[2], Marco Cifrodelli[2], Javier Estévez[1] and Alessia Flammini[2].

1 Engineering Projects Area, Department of Rural Engineering, University of Cordoba, Spain.

2 Department of Civil and Environmental Engineering, University of Perugia, Italy

* Corresponding author: amanda.garcia@uco.es

Abstract

Intensity Duration Frequency (IDF) curves are important tools for hydraulic and hydrologic design. Considering that there are different approaches to obtain the rainfall quantiles on which the IDF curves are based, the availability of a method to evaluate their reliability is of great importance. With this aim, in this work the multifractal properties of hourly rainfall data recorded at 23 rain gauges in the Umbria Region (Italy) have been used to select the most appropriate frequency analysis method of extreme annual rainfall at each location. Three methods have been tested: Local Frequency Analysis (by fitting at each station extreme annual rainfall data of different durations by a probability distribution function), Regional Frequency Analysis approach based on L-Moments and flood index method (considering the extreme annual rainfall data from all the stations), and a variant of the latter method known as In-site Regional Frequency Analysis (based on the consideration of the station as a region). Therefore, quantiles of rainfall for different durations and return periods have been obtained. These quantiles have been fitted to the Montana Intensity-Duration-Frequency (IDF) curve. The scaling properties of rainfall have been obtained through out the empirical moments scaling function *K(q).* Their comparison with some scaling behavior properties of the IDF curves has let the selection of the most adequate quantile estimation method at each site, being the Regional Frequency Analysis the most appropriate one for 14 out of the 23 sites included in the study.

Keywords: extreme rainfall, multifractality, frequency analysis methods.

1. Introduction

IDF curves are widely used in water resources management projects (Hajani and Rahman, 2018) to obtain the value of the design storm depth. They are derived by fitting extreme quantiles of rainfall (obtained from frequency analysis methods) by parametric equations characterized by different number of parameters like: the Montana equation widely used in real applications (Di Baldassarre et al., 2006a), the Temez equation proposed by the Spanish Water Authority Centre of Hydrographic Studies‑CEDEX (Témez, 1987), the four-parameter IDF function considered by Koutsoyiannis et al. (1998) or the one propose by Chow et al. (1988). All of these equations let to obtain the values of rainfall intensity, *i*, as a function of the duration, *D*, return period, *T* (or frequency), and some fitted parameters.

When observed rainfall extremes are available for a certain duration, the design storm value (quantile of rainfall with a certain duration and return period) can be estimated by fitting on such data a suitable extreme probability distribution function (e.g. Di Baldasarre et al., 2006a). For low return periods, short extreme data series are good enough to estimate quantile values, so that a local or at-site frequency analysis is valid. As the return period value increases, the length of the data series needs to enlarge. Long rainfall data series are usually available for daily durations, but not for shorter time periods. In this situation, a Regional Frequency Analysis shows up as a good option in order to increase the amount of available data and also to improve quantile estimates (e.g. Hosking and Wallis, 1997; Yu et al, 2004; García-Marín et al., 2011; Haddad et al, 2011; Du et al, 2014; Liu et al, 2015; Hajani and Rahman, 2018; Moujahid et al., 2018).

This approach solves the problem of lack of data in time with the abundance of data in space. The bigger the sample of data fitted by a probability distribution function the higher confidence is on estimated quantile values (specially for low frequencies of occurrence). A region will be composed with data from different sites that share the same probability distribution function.

An alternative approach that is called In-site regionalization consists of applying the regionalization technique to a specific station (e.g. De Salas and Fernández, 2007; Ayuso-Muñoz et al, 2015). Even in this case the amount of available data increases (that is the main aim of the Regional Frequency Analysis), but with data recorded in the same station over different durations. The region will be formed by series that are considered to come from similar frequency distribution as in the standard regionalization technique.

IDF curves have been widely analyzed all over the world (e. g. Jakob et al., 2007; Xu and Tung, 2009; Lee at al., 2010; Haddad et al., 2011; Dourte et al., 2013; Du et al., 2014) and different approaches belonging to the aforementioned techniques have been adopted to obtain them (Elsebaie, 2011; Mamoon et al., 2014; Liu et al., 2015). Some studies show comparisons between at site and regional estimates. Haddad et al. (2011) found that regional design rainfall estimates were generally greater than the at-site estimates. Moujahid et al., (2018) found an increase in events intensities derived from Regional Frequency Analysis. Hajani and Rahman (2018) compared IDF curves derived by different distributions and methods and found that the regional curves values were generally higher than the at-site IDF ones. Despite the existence of these works, a widely recognized method to evaluate the reliability of different quantile estimate approaches to obtain IDF curves that best reproduces the behavior of real extreme annual rainfall data in a certain place is still lacking.

Rainfall and Intensity-Duration-Frequency (IDF) curves satisfy scaling relations that are based on the complexity of rainfall that exhibits self-similarity at different scales and can be considered as fractal (e.g. Schertzer and Lovejoy, 1987; de Lima and Grasman, 1999; Kiely and Ivanova, 1999; Castro et al.,2004; Langousis et al., 2009; García-Marín et al., 2013; Valencia et al., 2010; Schertzer and Lovejoy, 2011; Rodríguez et al., 2013; Casas-Castillo et al., 2018). Self-similarity processes look the same regardless of the scale where they are observed. Fractal processes exhibit the same behavior for different scale measurements, so they are self-similar. Fractal self-similarity of rainfall has a statistical nature so that its scaling properties can be expressed by statistical relationships (Schertzer and Lovejoy, 1987; Schertzer and Lovejoy, 2011). Moreover, the probability distribution of the annual maximum rainfall intensities follows scale-relationships (Burlando and Rosso, 1996).

The scale invariance character of rainfall must be reproduced by any rainfall model. A relation between rainfall fractal behavior and IDF scaling exists as detailed in section 3.4 (e.g. Veneziano and Furcolo, 2002; García-Marín et al., 2013). Specifically, IDF values are simple scaled with a power law dependence on the duration (*D*) and return period (*T*). The power law exponent can be calculated from the moment scaling exponent function *K(q)* that characterizes the multifractality of rainfall time series (Veneziano and Furcolo, 2009).

As a novelty and based on the evidence that IDF models satisfy the scaling behavior of rainfall (e.g. Yu et al., 2004; Ghanmi et al, 2016; Rodríguez-Solá et al., 2017; Choi et al., 2018), the objective of this work is to use the multifractal analysis to select the most appropriate frequency analysis method to obtain rainfall quantiles at a certain place. With this purpose the multifractal characterization of hourly rainfall data series in 23 rain gauges stations in the Umbria Region (Italy) is performed. Afterwards, rainfall quantiles are obtained at each place by applying three different methods: Local, Regional and In-site Regional Frequency Analysis. The local or at site rainfall frequency analysis is applied by fitting extreme annual rainfall data registered at each station by the General Extreme Values (GEV) probability distribution function. The regional and In-site regional analysis performed are based on the regionalization methodology proposed by Hosking and Wallis (1997). For the former, the extreme annual rainfall annual data of several durations from all the available stations are used and regionalization is studied for each duration. For the latter, the regionalization is made for each site considering only its extreme annual rainfall data for the available durations. For each frequency analysis method, all the quantiles obtained are fitted by the Montana IDF model. The existing relation between the multifractality of rainfall data previously studied and the scale invariance of the IDF curves is finally used to select the most appropriate rainfall frequency analysis methodology at each place.

2. Study area and data source

In this study, rainfall data from the Umbria Region (central Italy) are used. The Umbria Region, with an extension of 8,456 Km^2, exhibits a mountainous landscape along its eastern side where Apennine Mountains reach up to 2,000 m.a.s.l, and a hilly morphology in the central and western zones with altitudes ranging from 100 to 800 m a.s.l. A wide part of the study area is included in the Tiber River basin, which crosses the Region from North to South-West.

Mean annual rainfall for the last century is about 900 mm, with values varying in space from 650 mm to 1450 mm. Based on 1921– 2015 period and a network of more than 90 rain gauges, the highest rainfall values usually take place during the autumn-winter seasons. The highest monthly rainfall values generally occur during the autumn-winter period, together with floods caused by widespread rainfall. The highest and lowest rainfall depths typically take place in November and July, respectively.

Over the past 15 years the region has been affected by five significant droughts (2001 to 2003, then in 2007, 2012, 2015 and 2017) as well as by six dangerous flood events (one occurred in 2005, one in 2008, one in 2010, two in 2012 and one in 2013) with very large impacts in economic terms (Morbidelli et al., 2018).

The study area is currently monitored through a dense rain gauge network (about 1 rain gauge every 90 Km^2) (Figure 1). In this study 23 rain gauge stations characterized by continuous hourly rainfall data from 1992 to 2015 are considered (Table 1 and Figure 1). One of the interests of the multifractal approaches is to use all the available data to extract the best information possible of the process under analysis. Therefore, for the multifractal approach, the continuous hourly rainfall data series are used, whereas for the quantile estimation the extreme annual rainfall data series (composed by the highest rainfall value in a year for a certain duration) are obtained for durations of 1, 3, 6, 12 and 24 hours.

3. Methodology

3.1. Multifractality

Fractal and multifractal approaches can be used for modelling time series and deriving predictions regarding extreme events. Multifractal analysis is applicable to variables self-similarly distributed on a geometric support that is represented by a line (i.e., time series), plane, volume, or fractal set. To identify multifractality in hydrological time series it can be assumed that the variability of the process under study can be modeled as a stochastic turbulent cascade process (Shertzer and Lovejoy , 1987; Gupta and Waymare, 1993; Over and Gupta, 1994; Lovejoy and Schertzer, 1995). A cascade process can be described as eddies breaking up into smaller sub-eddies, each of which receives a part of the flux from its parent body. This cascade process-type behavior can be used with rainfall data to transfer information from some temporal or spatial scales to another, if scale invariance is previously found in the data set.

To identify multifractality in rainfall data sets the statistical moments scaling method has been widely applied (Sivakumar, 2001). To perform the analysis, the time series has to be divided into non-overlapping intervals of a certain time resolution. The ratio of the field maximum scale to this interval is the scale ratio, λ. By this process, the time is scaled so that the duration of the longest period of interest is 1 (De Lima and Grasman, 1999). For a time interval *i* at the scale ratio λ, the mean rainfall intensity is given by $R(\lambda, i)$. In order to obtain non-dimensional values, the mean rainfall intensity $R(\lambda, i)$ has to be normalized by the so-called joint average of the mean rainfall intensities obtained for λ = 1 (the average found at the higher resolution), $\langle R(1, i)\rangle$, where $\langle\ \rangle = \left\{\frac{1}{N_\lambda}\sum_{j=1}^{\lambda}\ \right\}$, with N_λ the number of non-overlapping time intervals in which the time series is divided for a certain λ. The non-dimensional mean rainfall intensity for an interval *i* is then obtained as $\varepsilon\,(\lambda, i) = R(\lambda, i)/\langle R(1, i)\rangle$. The average q^{th} moments of the rainfall intensities of the process at resolution level λ, $\langle\varepsilon_\lambda^q\rangle$, can then be obtained and their scaling can be described by the *K(q)* function, that satisfies (Schertzer and Lovejoy, 1987; Lovejoy and Schertzer, 1990):

$$\langle\varepsilon_\lambda^q\rangle \approx \lambda^{K(q)} \qquad (1)$$

The scaling behavior given by equation 1 can be investigated by plotting $\langle\varepsilon_\lambda^q\rangle$ as a function of λ in a log-log plot diagram for several values of q. High and low values of q are related to extreme (very high or very low) values of rainfall. The former are conditioned by the length of data and the latter by the resolution of the pluviograph (commonly 0.1 mm). Therefore, a wide range of *q* moments values greater and lower 1 is recommended to describe the scale behavior of rainfall in a certain place (e.g. De Lima and Grasman, 1999). The linear fits of the log-log plot of equation 1 let to obtain the complete *K(q)* function and give information about the temporal scale invariance of the data set.

Different shapes of *K(q)* can be expected for mono and multifractal processes. For the former, *K(q)* versus q is a straight line, whereas for the latter a convex function appears (e.g. Yu et al., 2014). If K(q) is linear through the origin, the measure is self-similar. The value of *K(0)* is related to the zeros of the data series and also reflects the codimension of the field > 0 .

The codimension function $c(\gamma)$ can describe the probability distribution of the process intensity. It also indicates how the histograms of a variable change with resolution. It can be obtained parametrically as $c(q) = q\gamma(q) - K(q)$, where $\gamma(q) = dK(q)/dq$ (Parisi and Frish, 1985; Veneziano and Furcolo, 2002). The value of γ_{max}, which is the maximum value of the order of singularity γ, can provide information about the rare or extreme events in the data series (e.g. Veneziano and Furcolo, 2002).

3.2. Frequency analysis of extreme events

The main objective of frequency analysis is the estimation of extreme events corresponding to different return periods (quantiles) by using probability distribution functions. For this purpose, maximum annual rainfall data series are used, being composed by the highest annual values of rainfall for certain duration. For annual series, the return period of an event is the reciprocal of the probability of exceedance of that event (Koutsoyiannis et al., 1998; Langousis et al., 2009) and can be also defined as the average time interval between exceedances of a certain value.

For small return periods or long historical data series, single-site or Local Frequency Analysis (LFA) of extreme data is enough to obtain quantile values. When dealing with rainfall, quantiles are usually estimated to obtain IDF curves. For this purpose, in many studies GEV probability distribution function and its particular form Gumbel are applied to extreme rainfall (Menabde et al., 1999; Bougadis and Adamowski, 2006; Gubareva and Gartsman, 2010; Ghanmi et al., 2016; Choi et al., 2018). The GEV function is preferred for rainfall data series with high extreme values (e.g. Coles, 2001; Koutsoyiannis, 2004; Russell, 2019).

Regardless the probability distribution function used for an at-site frequency analysis, the main limitation of this analysis appears when the objective is to estimate extreme rainfall quantiles for high return periods starting from short-length data series. Being this kind of data the most frequently found all over the world, the Regional Frequency Analysis (RFA) appears then as a useful tool to solve the problem of temporal data scarcity by increasing data through the space (e.g. Rostami, 2013).

The RFA methodology used in this work was proposed by Hosking and Wallis (1997) and is based on L-moments and the Flood Index method (Dalrymple, 1960). The L-moments introduced by Hosking (1990, 1992) are commonly used in RFA of rainfall data (e.g. Yang et al., 2010; Zakaria and Shabri, 2012; Monjahid et al., 2018). They are linear functions of the probability weighted moments (Greenwood et al., 1979). For a sample size *n*, the estimator of probability weighted moments is given by $b_r = n^{-1}\binom{n-1}{r}^{-1}\sum_{j=r+1}^{n}\binom{j-1}{r}x_{j:n}.$, being *x* the variable under analysis. The sample L-moments are obtained as linear combinations of b_r, being $l_1 = b_0$, $l_2 = 2b_1 - b_0$, $l_3 = 6b_2 - 6b_1 + b_0$, $l_4 = 20b_3 - 30b_2 + 12b_1 - b_0$, among others. The L-moments ratios are given by $t_r = l_r/l_2$ (for *r* = 3, 4…). The value of $t = l_2/l_1$ is known as L-coefficient of variation. Flood index procedures are a convenient way of pooling summary statistics from different data samples. The term "flood index" arose because early applications of the procedure were performed on flood data in hydrology (e.g., Dalrymple. 1960), but the method can be used with any kind of data. The key assumption of a flood index procedure is that a group of sites forms a homogeneous region, that is their data sets are characterized by frequency distributions identical apart from a site-specific scaling factor, called flood index.

The RFA is advantageous over at-site analysis if homogeneous regions can be compound with the available data series. A homogeneous region is composed by stations that have identical frequency distributions apart from a site-specific scale factor. The obtaining of homogenous regions is the most critical step in RFA and several methodologies and site-characteristics can be applied to group stations into homogeneous regions (e.g. García-Marín et al., 2011, 2015; Medina-Cobo et al., 2017; Satyanarayana and Srinivas, 2011).

In order to easily understand the main steps of RFA, appendix A can be seen. As a first step in RFA, L-moments and their corresponding L-moments ratios (L-coefficient of variation ($t^{(i)}$), L-skewness ($t_3^{(i)}$) and L-kurtosis ($t_4^{(i)}$)) have to be obtained for all the data series (for each site or station, *i*) used in the analysis (notice that (*i*) changes to (R) if we refer to the region instead of the site).The three L-moments ratios of a certain site are considered as components of a vector in a three-dimensional space with L-moments ratios as coordinates.

Considering that each site is then characterized by a vector, the whole set of stations forms a cloud of points in that space. Any point far from the center of that cloud (being a point with average L-moments ratios values as coordinates) has to be considered as discordant and excluded from the analysis. Specifically, for each station the discordancy, D_i, with respect to the center can be determined as (Hosking and Wallis, 1997):

$$D_i = \frac{1}{3} N (u_i - \bar{u})^T A^{-1} (u_i - \bar{u}) \tag{2}$$

where $u_i = (t^{(i)}, t_3^{(i)}, t_4^{(i)})$, $\bar{u} = N^{-1} \sum_{i=1}^{N} u_i$, $A = \sum_{i=1}^{N} (u_i - \bar{u})(u_i - \bar{u})$ and N is the number of stations. If the obtained D_i value exceeds a critical value that depends on the number of sites and can be found in Hosking and Wallis, 1997, the site has to be excluded from the analysis because is discordant. The critical values of discordancy vary from 1.333 for regions of five sites to 3.000 for regions of fifteen or more sites (Hosking and Wallis, 1997).

In order to test if a group of sites forms a homogeneous region, the heterogeneity measure *H*, that compares the between-site variations in sample L-moments for a group of sites with what would be expected for a homogeneous region, has to be calculated. For that purpose, it is necessary:

- to calculate the weighted standard deviation of at-site sample L-coefficient of variation, *V*, given by $V = \left\{ \sum_{i=1}^{N} n_i (t^{(i)} - t^R)^2 / \sum_{i=1}^{N} n_i \right\}^{1/2}$, with $t^R = \sum_{i=1}^{N} n_i t^{(i)} / \sum_{i=1}^{N} n_i$, *n* the number of data, and $t^{(i)}$ the L-coefficient of variation of the station;
- to fit a kappa distribution to the regional average L-moments ratios;

- to simulate a large number, N_{sim}, of realizations of a region with N sites, each having this kappa distribution as its frequency distribution; the simulated regions are homogeneous and the sites have the same record lengths as their real-world counterparts;
- to calculate V for each simulated region;
- to determine the mean, μ_V, and standard deviation, σ_V, of N_{sim} values of V from the simulations.

The heterogeneity measure can finally be obtained as:

$$H = \frac{(V - \mu_V)}{\sigma_V} \quad (3)$$

The region under consideration is acceptably homogeneous when $H<1$; possibly heterogeneous when $1<H<2$, and definitely heterogeneous for $H>2$. Once the homogeneity of a region is checked, the quantiles of the variable analyzed for several return periods can be obtained. Different probability distribution functions can be tested in order to select the most appropriate to describe the region and to obtain the regional growth curve, *q(F),* where *F* is the frequency. In a set of three-parameter candidate distributions, to measure the fit goodness of each one, the following statistics can be calculated:

$$Z^{DIST} = (\tau_4^{DIST} - t_4^R + B_4)/\sigma_4 \quad (4)$$

where τ_4^{DIST} is the L-kurtosis coefficient of the three-parameter distribution; t_4^R the regional average L-kurtosis coefficient; σ_4 the standard deviation of t_4^R which can be obtained by repeated simulation of a homogeneous region whose sites have the selected three-parameter frequency distribution and record lengths the same as those of the observed data; and B_4 is the bias in the regional average

L-kurtosis for regions with the same number of sites and the same record lengths as the observed data.

The fit of a specific distribution is considered to be adequate if the value of statistics $|Z^{DIST}|$ is below or the same as 1.64 at the significance degree of 90%.

By applying the flood index method, the quantiles Q_i at a site i can then be obtained by,

$$Q_i(F) = \mu_i q(F), i = 1...N \tag{5}$$

being μ_i the flood index (average of data at site i).

Besides the lack of long data series, in some cases there is a lack of spatial information, so a regular RFA cannot be performed. In this case, the regionalization technique can be applied in a single station (de Salas and Fernández, 2007). The station now becomes the region, and the sites of the region are the available data series of different duration. This method is called the In-site Regional Frequency Analysis (IRFA) and can be applied at any location where rainfall records of around 10 min to 24 h duration are available, allowing the development of robust quantiles estimations (Ayuso-Muñoz et al., 2015). The following steps to be followed are the same as stated before.

3.3. IDF formulation

Regardless of the statistical frequency analysis performed, once the quantiles of rainfall are obtained for different durations, an IDF model can be fitted. The most widely used approach consists of using a parametric model characterized typically from 2 to 4 parameters. As the number of parameters increases, the uncertainty of the estimations is amplified as well (Di Baldassarre et al., 2006a, b).

Several IDF equations can be found (e.g. Chow, 1964; Bell, 1969; Chen, 1983; García-Bartual and Schneider, 2001), being the one known as Montana Curve one of the most widely used (Di Baldassarre et al., 2006a):

$$i(T,D) = aD^{b-1} \qquad (6)$$

where a and b are the parameters that depend on the return period.

This equation shows some limitations for describing the behavior of short-term storms of less than 1 hour (Di Baldassarre et al., 2006a). Nevertheless, considering that it is widely used in the region of the present study and that the work is focused on durations higher than 1 hour, it has been chosen as IDF model.

3.4. Fractals and IDF

As Veneziano and Furcolo (2002) stated, most of the models of IDF curves belongs to self-similar models that satisfy simple-scaling relations, or asymptotically self-similar models. While the IDF curves satisfy simple scaling relations, temporal rainfall has multifractal scale invariance.

In engineering practice, the scaling relations between some IDF models and some multifractal parameters can be relevant. They can be used to obtain probable maximum precipitation estimates (Casas-Castillo et al., 2018), rainfall values of low durations by downscaling (Rodríguez-Solá et al., 2017) or rainfall values in un-gauged sites (Ghanmi et al., 2016).

One can consider:

$$i(T, D) = D^{-h} i(T, 1)$$

(7)

being h the self-similarity index, D the duration, $i(T, D)$ the intensity value with probability $1/T$, and T the return period.

On the other hand, in approximation and for large T:

$$i(T, 1) \propto T^{\alpha} \qquad (8)$$

where α is a constant (Bendjoudi et al., 1997; 1999).

For any finite range of T and $D \to 0$, Veneziano and Furcolo (2002) obtained that $h = \gamma_1$ and $\alpha = 1/q_1$, where γ_1 is the value of γ where the codimension function $c(\gamma)$ equals 1 and q_1 is the associated moment order.

The scaling of IDF can be then expressed by:

$$i(T, D) \propto D^{-\gamma_1} T^{1/q_1} \qquad (9)$$

The parameters γ_1 and q_1 can be obtained from $K(q)$ function (from the linear behavior of the field for q values greater than a given moment) and be related to the slope of the IDF obtained for a certain location. Thus, for each return period analyzed, the absolute value of the IDF slopes ($|b-1|$, equation (6)) should be close to the γ_1 value obtained from the multifractal analysis of rainfall data at the same place. Moreover, the slope of the line obtained from plotting the mean rainfall intensity values of several duration for different return periods should be close to the value $1/q_1$ (García-Marín et al., 2013).

4. Results and discussion

4.1. Mutifractal analysis of hourly rainfall in the Umbria Region

To obtain the empirical moments scaling exponent function $K(q)$ for the hourly rainfall data series in the 23 rain gauge stations available (Table 1), the log-log plot of the q^{th} average moments of the rainfall intensity ε_λ against the scale ratio λ has to be obtained at each place. Figure 2 shows this plot for three sites: Casacastalda, San Benedetto Vecchio and San Silvestro. As it can be observed, straight lines appear for moments higher (Figure 2a) and lower (Figure 2b) than 1. These straight lines give information about the scaling behavior of the moments observed in the range from 1 hour to 21 days for the three sites. Similar values have been found by other authors for different locations (e.g. Ladoy et al., 1991, 1993; Fraedrich and Larnder, 1993; Svensson et al., 1996; Tessier et al., 1996; Labat et al., 2002; García-Marín et al., 2013; Rodriguez et al., 2013). The same scale invariance regimens have been found for the hourly rainfall data of the other stations in the Umbria region.

The empirical function *K(q)* has been then obtained for the scale regimen detected and for all the sites. In Figure 3 the function K(q) for the same stations of Figure 2 is shown. In all three cases it shows a convex shape that gives information about the multifractal behavior of hourly rainfall at the Umbria region. Important information for the present work can be obtained from *K(q)* function, like the singularity values γ_1. The value for this parameter varies from 0.7219 for Casacastalda, to 0.7945 for San Benedetto Vecchio, with a value of 0.7491 for San Silvestro. Table 2 shows the values of γ_1 obtained from the scaling functions at all the sites analyzed, found to vary from 0.7125 to 0.8242. Similar results have been found by García-Marín et al. (2013) and Rodríguez-Solá et al. (2017) for rainfall data in Spanish stations. The value of moment order q_1 associated to γ_1 is also shown in Table 2 for all the sites, and in Figure 3 for the selected stations.

4.2. Quantile estimation

For the 23 rain gauges, the extreme annual rainfall data series are obtained by selecting the maximum rainfall value of certain duration (for durations of 1, 3, 6, 12 and 24 hours) for each year of data. The three different frequency analysis methods described in section 3 have been applied. Firstly, a LFA for each extreme data series at each location. Secondly, a RFA of extreme rainfall data for each duration considering all the sites. And finally, an IRFA at each location.

For the local analysis the GEV probability distribution function has been used, obtaining quantiles values of extreme rainfall for return periods of 5, 10, 25, 50, 100 and 200 years.

For each duration, a RFA has been performed (RFA_{ih} with i = 1, 3, 6, 12 and 24 hours) obtaining the results summarized in Table 3. As first step, the existence of discordant sites was checked by obtaining the values of D_i (equation 2). For the hourly extreme rainfall Regional Frequency Analysis (RFA_{1h}), only Petrelle station was found discordant, and was taken out of the analysis. Two sites resulted discordant for RFA_{3h}, Compignano and San Biagio della Valle; and two more for the RFA_{12h}, being Narni Scalo and San Benedetto Vecchio. For RFA_{6h} and RFA_{24h} no sites were found to be discordant.

The heterogeneity measure of the region compound by the remaining stations was then evaluated with equation 3. The values of H were found lower than one for RFA_{1h} (-0.11), RFA_{3h} (0.90), RFA_{12h} (0.54) and RFA_{24h} (0.97). These results show the homogeneity of the regions constituted by the considered stations for the selected durations. For RFA_{6h} the initial value of H was 1.96, which indicated that the region was probably heterogeneous and so it was divided into sub-regions. Following the methodology proposed by García-Marín et al. (2015), the multifractal characteristics of rainfall data were used to divide the

RFA_{6h} region into two regions. With the values of γ_1 and *K(0)* (Table 2) a K-means cluster analysis was performed and two groups of sites were obtained. Two new Regional Frequency Analyses were done, RFA_{6hA} and RFA_{6hB}. The first group was initially composed by 9 sites, whereas 14 stations formed the second. Petrelle station was discordant again, so finally two groups stayed, one with 8 sites and one with 14 stations (Table 3). Both groups were found to be homogeneous according to the values of *H* parameter, being -0.23 for RFA_{6hA} and -0.83 for RFA_{6hB}, respectively.

Once the homogeneous regions have been compounded, the growth curves are necessary in order to finally obtain the quantiles at all the sites. For this purpose, the goodness of a probability distribution function (pdf) in fitting the regional data has to be studied through the value of Z^{DIST} (equation 4). Five three-parameter probability distribution functions were tested, being the Generalized Logistic (GEN-LOG), Generalized Extreme Value (GEV), Generalized Normal (GEN-NOR), Pearson type III (PT-III) and Generalized Pareto (GEN-PAR).

Table 4 shows the values of statistics Z^{DIST} obtained by applying equation 4 for all the homogeneous regions and probability distribution functions. According to these results, the most appropriate function for each region is the one that gives the lowest value of $|Z^{DIST}|$ between 0 and 1.64. Based on that, the GEV pdf is the most appropriate for RFA_{3h}, RFA_{6hA} and RFA_{24h}, the GEN-NOR pdf is the best one for RFA_{6hB} and RFA_{12h}, whereas GEN-LOG is the most suitable for RFA_{1h}. The regional growth curves for each region can then be obtained with the selected probability distribution functions for several return periods (Table 5). For each duration, considering the calculated growth curve and the average datum at each site, the quantiles for different return periods were obtained by applying the flood index method (equation 5).

The IRFA was then applied at each location, considered as one region and each series of a particular duration (5, 10, 15, 20, 30 and 40 min, 1, 3, 6, 12 and 24 h) as one site into the region.

As it can be seen in Table 6, for 15 out of 23 In-site regions, all durations (sites) formed a homogeneous region with *H* values less than 1.00. For the In-site regions Forsivo, Gubbio and La Cima, some sites were found to be discordant and were removed from the analysis: 5’ and 24 h in Forsivo, 24 h in Gubbio and 5’, 10’, 15’, 20’ in La Cima. Once the discordant sites were eliminated, the In-site regions became homogeneous according to the *H* values (Table 6). In-site regions of Montelovesco and San Biagio della Valle, formed a homogeneous region considering all sites, with *H* values of -0.57 and -2.07, respectively. Nevertheless, no probability distribution function was found as a good candidate to fit the In-site regional data, according to the values of Z^{DIST} obtained. For Montelovesco two subregions were formed, one for durations less than one hour and another for durations greater than one hour (Table 6). Both of them were homogeneous. Petrelle In-site region was heterogeneous considering all sites, so two subregions were formed, and homogeneity results were obtained (Table 6). For San Biagio della Valle, some sites were removed in order to form a homogeneous region. The final In-site homogeneous region was compounded by 7 sites from 30’ to 24 h. Similar situation occurred for San Benedetto Vecchio station, where a final homogeneous region was formed removing some sites (Table 6).

The regional growth curves for each In-site region were then obtained with the selected probability distribution functions for several return periods (Table 7). As an example, and in order to compare the different approaches, the local, regional and in-site regional quantiles for 24 hours and 50 years of return period are shown in Figure 4. Notice that the values are missing for Forsivo and Gubbio, where 24 h duration was removed from the In-site Regional Frequency Analysis. As it can be seen in this figure, there is no a general pattern in the values obtained by the different methodologies. For 8 sites out of 23 (34.8%) local quantiles are the highest, for 10 sites out of 23 (43.5%) regional quantiles are the highest, whereas In-site quantiles are the highest only for 4 out of 23 sites (17.4%). In any case, the different values of quantiles obtained by the three applied methods provide different IDF curves and different design storm values. Thus, it is important to select the most appropriate IDF curve for each site in order to reproduce adequately the extreme rainfall behavior.

4.3. IDF fitting

All the quantiles of more than one hour of duration obtained by using LFA, RFA and IRFA have been fitted by the Montana IDF model (equation 7). As an example, Table 8 shows the values of parameters *a* and *b* for Casacastalda, San Benedetto Vecchio and San Silvestro. It can be noticed that, for each site and frequency analysis, the parameter *b* has a constant value for all the return periods. This is because when fitting a different *b* for each T_i (i = 5,…, 200 years) value, it was observed that, in some cases, $i_{D,T_2} < i_{D,T_1}$, being $T_2 > T_1$. In order to avoid that situation, a constant value of *b* was obtained as the average of all the *b* values independently estimated for the different return periods.

4.4. Selection of frequency analysis

The scale invariance properties of IDF curves and their relationship with multifractal parameters (e.g. Veneziano and Furcolo, 2002; García-Marín et al., 2013) are used in this work to select the most appropriate frequency analysis at each site of the Umbria region.

In this context, for each site, the absolute value of slopes of the estimated IDF lines $Slope_{IDF}$ has to be close to the value obtained for the singularity value γ_1 in the multifractal analysis of the hourly rainfall data (Table 2). For Casacastalda, San Benedetto Vecchio and San Silvestro, Figure 5a shows $Slope_{IDF}$ for each approach and the singularity value γ_1 obtained for each location. As it can be checked, for Casacastalda the values obtained for LFA and IRFA are lower than γ_1 (0.7219), being the value for Regional Frequency Analysis very close to the singularity value. For San Benedetto Vecchio station, the closest value to γ_1 (0.7945) is that obtained through LFA, whereas at San Silvestro station the IRFA is the most suitable to reproduce the γ_1 value (0.7491).

For the same stations, Figure 5b shows the slope of the fit of the rainfall intensity values averaged over durations for different return periods ($Slope_{ARI}$). The slope value has to be close to the value of $1/q_1$ obtained for each location (Table 2). It can be observed that for Casacastalda the closest value to $1/q_1$ (0.2222) is the one obtained with RFA, being the results obtained with LFA and IRFA the worst. In San Benedetto Vecchio station, the best result is obtained with LFA, whereas IRFA is the best to fit $1/q_1$ (0.2667) in San Silvestro station.

According to the results described above, the IDF curve obtained with quantiles from RFA is the best for Casacastalda station, the one obtained with Local Frequency Analysis quantiles is the most appropriate for San Benedetto Vecchio station, and the IDF curve obtained with quantiles from IRFA is the best choice for San Silvestro station.

For the rest of stations, the $|\text{Slope}_{\text{IDF}}|$ and the $\text{Slope}_{\text{ARI}}$ values obtained for the three approaches are shown in Table 9. The values in Table 9 have to be compared to those in Table 2 (γ_1, q_1) in order to select the best IDF curve at each location.

To better synthetize the results of Figure 5 and Table 9, the values of $(\gamma_1 - Slope_{IDF})^2$ and $|Slope_{ARI} - (1/q_{1)}|$ have been obtained for each site and frequency analysis and shown in Table 10 (e.g. García-Marín et al., 2013).

For each multifractal criterium the green values indicate when the LFA is the best, the red values have the same meaning but for RFA, and the blue ones correspond to the best results for IRFA. Combining both multifractal criteria, a decision can be made and a proper frequency analysis can be selected for each site. The selected frequency analysis at each location is shown in bold. Table 10 shows that only for three sites (Città di Castello, Ripalvella and San Benedetto Vecchio stations) the LFA is the best option. IRFA exhibits to be the best choice only at San Silvestro station. RFA is the most appropriate frequency analysis for 14 out of 23 sites. For the rest of stations (5), very close results have been obtained for all frequency analyses, but no coincidence of the two multifractal criteria has been found.

5. Conclusions

The scaling properties of IDF curves are used in this work to select the proper frequency analysis method to obtain quantiles of rainfall in the Umbria Region (Italy). With this purpose, rainfall data series from 23 rainfall gauges were used.

The multifractal properties of hourly rainfall were evaluated by applying the statistical moments scaling method. The empirical function $K(q)$ was obtained at each place, and some important multifractal parameters were identified: γ_1, q_1 and the value of $K(0)$.

Three frequency analyses of annual maximum rainfall data for different durations have been considered: local, regional and In-site regional frequency analyses. For the Local Frequency Analysis, the GEV probability distribution function was used, and the local quantiles were obtained. The Regional Frequency Analysis proposed by Hosking and Wallis (1997) was performed for each duration, and homogeneous regions of annual maximum rainfall were composed. Different probability distribution functions were tested at each region in order to obtain the regional growth curves and the corresponding quantiles at each location.

The Regional Frequency Analysis was also applied at each station (In-site Regional Frequency Analysis), considering the site as a region and the maximum annual rainfall data series as the sites of the region, and the quantiles were derived. Different quantile values were obtained at each station for the different frequency analyses approaches, with no regular pattern for the highest or lowest values for a certain approach.

The rainfall quantiles obtained through the different approaches were fitted by the Montana IDF curve. Then, following the theory proposed by Veneziano and Furcolo (2002), the values of γ_1 were compared with the absolute value of slopes of the different IDF curves, and the values of $1/q_1$ were compared to the slope of mean rainfall intensity versus return period fit. These comparisons between multifractal parameters and IDF properties let to select the most appropriate frequency analysis at each location. The Regional Frequency Analysis gave the best results for the 61% of stations, closely followed by the Local Frequency Analysis that was the best option for the 13% of sites. The In site Regional Frequency Analysis was the most appropriate only in one station, and for five more similar values were obtained with the three frequency analysis approaches.

Thus, the analysis of the scaling behavior of rainfall proposed here seems to be a good tool to decide which frequency analysis is adequate at a certain place. Lastly, the scaling behavior of rainfall can be analyzed through data sets with time resolution that can vary from fine (e.g. Veneziano and Furcolo 2002; Rodríguez-Solá et al., 2017) to coarse (e.g. Garcia-Marín et al., 2015; Casas-Castillo et al., 2018). When scale invariance is found in a data set, the downscaling or upscaling of maximum rainfall data can be performed. This last procedure let to obtain both quantiles and IDF relations at places where no fine time resolution data are available. Therefore, different types of quantile estimation can be always obtained and the methodology proposed in this work can be useful to decide among them.

APPENDIX A

Figure A.1 shows a flowchart that can summarizes the main steps to be followed in RFA. Let suppose a potential region (REGION 1 in Figure 6) composed by *n* sites with extreme annual rainfall data series. Calculate the L-moments of the data series. Afterwards, obtain the values of discordancy D_i (from equation 2). Remove from the analysis all the *m* sites with discordancy values greater than the critical one, and repeat this procedure until no discordant site is found. With all the non-discordant sites, calculate the heterogeneity value (*H*) by applying equation 3. If *H* value is greater than one, star the process again subdividing REGION 1 into sub-regions. If the *H* value is lower than one, the region can be considered as homogeneous and the value of Z^{DIST} for a number of candidate probability distribution functions (p.d.f.) has to be obtained by applying equation 4. If the absolute value of Z^{DIST} is lower than 1.64 all the data of REGION 1 can be fitted by a p.d.f. (consider the one with the lowest $|Z^{DIST}|$) and the regional growth curve can be obtained. On the contrary, fit the data through the Wakeby p.d.f. Finally, obtain the quantile values Q_i by applying equation 5.

AKLOWLEDGEMENTS

A.P. García-Marín acknowledges the support from the Spanish Ministry of Science, Innovation and Universities with the project AGL2017-87658-R. She also acknowledges the collaboration and hosting of the University of Perugia and specifically of the Hydrology team of the Department of Civil and Environmental Engineering.

6. References

Ayuso-Muñoz, J. L., García-Marín, A. P., Ayuso-Ruíz P., Estévez, J., Pizarro-Tapia, R., Taguas, E. V., 2015. A More Efficient Rainfall Intensity-Duration-Frequency Relationship by Using an “at-site” Regional Frequency Analysis:

Application at Mediterranean Climate Locations. Water Resources Management, DOI 10.1007/s11269-015-0993-z.

Bell, F. C., 1969. Generalized rainfall-duration-frequency relationships. Journal of Hydraulics Engineering 95(HY1): 311–327.

Benjoudi, H., Hubert, P., Schertzer, D., Lovejoy, S., 1997. Interpretation multi-fractale des courbes intensite´-duree-frequance des precipitations, Geosci. Surface Hydrol. 325, 323–336.

Benjoudi, H., Hubert, P., Schertzer, D., Lovejoy, S., 1999. Multifractal explanation of rainfall intensity-duration-frequency curves, paper presented at EGS 24th General Assembly, Eur. Geophys. Soc., The Hague, Nether- lands, 19-23 April.

Bougadis, J., Adamowski, K., 2006. Scaling model of a rainfall intensity–duration–frequency relationship. Hydrological Processes, 20, 3747–3757. https://doi.org/10.1002/hyp.6386.

Burlando, P, Rosso, R., 1996. Scaling and multiscaling models of deph-duration-frequency curves for storm precipitation. J Hydrol 187:45– 64.

Casas-Castillo, M.C., Rodríguez-Solá, R., Navarro, X., Russo, B, Lastra, A., González, P., Redaño, A., 2018. On the consideration of scaling properties of extreme rainfall in Madrid (Spain) for developing a generalized intensity-duration-frequency equation and assessing probable maximum precipitation estimates. Theor Appl Climatol, 131:573–580.

Castro, J., Cârsteanu, A., Flores, C. 2004. Intensity-duration-area frequency functions for precipitation in a multifractal framework. Physica A: Statistical Mechanics and its Applications 338(1–2): 206–210.

Chen, C., 1983. Rainfall intensity-duration-frequency formulas. Journal of Hydraulics Engineering 109(12): 1603–1621.

Choi, J., Lee, O., Jang, J., Jang, S., Kim, S., 2018. Future intensity–depth–frequency curves estimation in Korea under representative con- centration pathway scenarios of Fifth assessment report using scale-invariance method. Int J Climatol. 1–14. https://doi.org/10.1002/joc.5850.

Chow, V.T., 1964. Handbook of Applied Hydrology. McGraw- Hill: New York.

Chow, V.T., Maidment, D.R., Mays, L.W. 1988. Applied Hydrology. McGraw-Hill: New York.

Coles, S., 2001. An introduction to statistical modelling of extreme value. Springer, London.

Dalrymple, T., 1960. Flood frequency analysis. USGS, Water Supply Paper 1543-A.

de Lima, M.I.P., Grasman, J. 1999. Multifractal analysis of 15-min and daily rainfall from a semi-arid region in Portugal. Journal of Hydrology 220: 1–11.

de Salas, L., Fernández, J.A., 2007. “In-site” regionalization to estimate an intensity-duration-frequency law: a solution to scarce spatial data in Spain. *Hydrol. Process.* **21**, 3507–3513.

Di Baldassarre, G., Brath, A., Montanari, A., 2006a. Reliability of different depth-duration-frequency equations for estimating short-duration design storms. Water Resources Research 42: W12501. DOI: 10.1029/ 2006WR004911.

Di Baldassarre, G., Castellarin, G.A., Brath, A., 2006b. Relationships between statistics of rainfall extremes and mean annual precipitation: an application for design-storm estimation in northern central Italy. Hydrology Earth System Sciences 10: 589–601.

Dourte, D., Shukla, S., Singh, P., Haman, D., 2013. Rainfall intensity duration frequency relationships for Andhra Pradesh, India: changing rainfall patterns and implications for runoff and groundwater re- charge. J Hydrol Eng 18(3):324–330.

Du, H., Xia, J., Zeng, S., 2014. Regional Frequency Analysis of extreme precipitation and its spatio-temporal characteristics in the Huai River Basin, China. Nat Hazards 70:195–215.

Elsebaie, I. H., 2011. Developing rainfall intensity duration frequency relationship for two regions in Saudi Arabia. J King Saud Univ Eng Sci 24:131–140.

Fraedrich, K., Larnder, C., 1993. Scaling regimes of composite rainfall time series. Tellus Series A-Dynamic Meteorology and Oceanography 45A: 289–298.

García-Bartual, R., Schneider, M., 2001. Estimating maximum expected short-duration rainfall intensities from extreme convective storms. Physics and Chemistry of the Earth B 26: 675–681.

García-Marín, A. P., Estévez, J., Medina-Cobo, M. T., Ayuso-Muñoz, J. L., 2015. Delimiting homogeneous regions using the multifractal properties of validated rainfall data series. J Hydrol 529:106–119.

García-Marín, A. P., Ayuso-Muñoz, J. L., Taguas-Ruiz, E. V., Estévez, J., 2011. Regional analysis of the annual maximum daily rainfall in the province of

Málaga (southern Spain) using the principal component analysis. Water Environ. J. 25 (4) 522-531.

García-Marín, A. P., Ayuso-Muñoz, J. L., Jiménez-Hornero, F. J., Estévez, J., 2013. Selecting the best IDF model by using the multifractal approach. Hydrol. Process. 27, 433–443.

Ghanmi, H., Bargaoui, Z., Mallet, C., 2016. Estimation of intensity–duration–frequency relationships according to the property of scale invariance and regionalization analysis in a Mediterranean coastal area. Journal of Hydrology, 541, 38–49. https://doi.org/10.1016/j.jhydrol.2016.07.002.

Greenwood, J.A., Landwehr, J.M., Matalas, N.C., Wallis, J.R., 1979. Probability weighted moments: définition and relation to parameters of several distributions expressible in Inverse form. Water Resour. Res. 15, 1049–1054.

Gubareva, T.S., Gartsman, B.I., 2010. Estimating distribution parameters of extreme hydrometeorological characteristics by l-moment method. Water Resour. 37 (4), 437–445. http://dx.doi.org/10.1134/S0097807810040020.

Haddad, K., Rahman, A., Green, J., Kuczera, J., 2011. Design Rainfall Estimation for Short Storm Durations Using L-Moments and Generalized Least Squares Regression-Application to Australian Data. International Journal of Water Resources and Arid Environments 1(3): 210-218.

Hajani, E., Rahman, A., 2018. Design rainfall estimation: comparison between GEV and LP3 distributions and at-site and regional estimates. Nat Hazards 93:67–88.

Hosking, J. R. M., 1992. Moments or L-moments? An example comparing two measures of distributional shape. The American Statistician 46(3):186–189.

Hosking, J. R. M., Wallis, J. R., 1997. Regional Frequency Analysis. An Approach Based on L-Moments. Cambridge. Cambridge University Press.

Hosking, J. R. M., 1990. L-moments: Analysis and Estimation of Distributions using Linear Combinations of Order Statistics. Journal of the Royal Statistical Society B 52(1): 105–124.

Jakob, D., Xuereb, K., Taylor, B., 2007. Revision of design rainfalls over Australia: a pilot study. Aust J Water Resour 11(2):153–159.

Kiely, G., Ivanova, K. 1999. Multifractal analysis of hourly precipitation. Physics and Chemistry of the Earth Part B—Hydrology Oceans and Atmosphere **24**: 781–786.

Koutsoyiannis, D., Kozonis, D., Manetas, A., 1998. A mathematical framework for studying rainfall intensity duration frequency relation-ships. Journal of Hydrology 206: 118–135.

Koutsoyiannis, D., 2004. Statistics of extremes and estimation of extreme rainfall: II. Empirical investigation of long rainfall records/Statistiques de valeurs extrêmeset estimation de précipitations extrêmes: II. Recherche empirique sur de longues séries de précipitations. Hydrological Sciences Journal, 49:4, -610, DOI: 10.1623/hysj.49.4.591.54424.

Labat, D., Mangin, A., Ababou, R., 2002. Rainfall-runoffs relations for karstic springs: multifractal analyses. Journal of Hydrology 256: 176–195.

Ladoy, P., Lovejoy, S., Schertzer, D., 1991. Extreme variability of climatological data: Scaling and intermittency. In Non-linear variability in Geophysics, Schertzer D, Lovejoy S (eds). Kluwer Acad.: Norwell, Mass; 241–250.

Ladoy, P., Schmitt, F., Schertzer, D., Lovejoy, S., 1993. The multifractal temporal variability of Nimes rainfall data. Comptes Rendus de l` Academie des Sciences Serie II 317(6): 775–782.

Langousis, A., Veneziano, D., Furcolo, P., Lepore, Ch., 2009. Multifractal rainfall extremes: theoretical analysis and practical estimation. Chaos, Solitons and Fractals 39: 1182–1194.

Lee, C., Kim, T., Chung, G., Choi, M., Yoo, C., 2010. Application of bivariate frequency analysis to the derivation of rainfall frequency curves. Stoch Env Res Risk Assess 24:389–397.

Liu, J., Doan, C., Liong, S., Sanders, R., Dao, A., Fewtrell, T., 2015. Regional Frequency Analysis of extreme rainfall events in Jakarta. Nat Hazards 75:1075–1104.

Lovejoy, S., Schertzer, D., 1990. Multifractals, universality classes, satellite and radar measurements of clouds and rain. Journal of Geophysical Research 95(D3): 2021–2034.

Mamoon, A., Joergensen, N. E., Rahman, A., Qasem, H., 2014. Derivation of new design rainfall in Qatar using L-moment based index frequency approach. Int J Sustain Built Environ 3:111–118.

Medina-Cobo, M. T., García-Marín, A. P., Estévez, J., Jiménez-Hornero, F.J., Ayuso-Muñoz, J.L., 2017. Obtaining Homogeneous Regions by Determining the Generalized Fractal Dimensions of Validated Daily Rainfall Data Sets. Water Resouces Management, 31: 2333-2348.

Menabde, M., Seed, A., Pegram, G., 1999. A simple scaling model for extreme rainfall. Water Resources Research, 35, 335–339.

Monjahid, M., Stour, L., Agoumi, A., Saidi, A., 2018. Regional approach for the analysis of annual maximum daily precipitation in northern Morocco. Weather and Climate Extremes, 21: 43-51.

Morbidelli, R., Saltalippi, C., Flammini, A., Corradini, C., Wilkinson, S. M., Fowler, H. J., 2018. Influence of temporal data aggregation on trend estimation for intense rainfall. Advances in Water Resources, 122: 304-316.

Moujahid, M., Stour, L., Agoumi, A., Saidi, A., 2018. Regional approach for the analysis of annual maximum daily precipitation T in northern Morocco. Weather and Climate Extremes 21 (2018) 43–51.

Parisi, G., Frish, U., 1985. A multifractal model of intermittency. In: Ghill M, Benzi R, Parisi G, editors. Turbulence and predictability in geophysical fluid dynamics. North Holland.

Rodríguez-Solá, R., Casas-Castillo, M. C., Redaño, A., 2013. Multifractal analysis of the rainfall time distribution on the metropolitan area of Barcelona (Spain). Meteorology and Atmospheric Physics 121(3–4): 181–187.

Rodríguez-Solá, R., Casas-Castillo, M.C., Navarro, X., Redaño, A., 2017. A study of the scaling properties of rainfall in Spain and its appropriateness to generate intensity–duration–frequency curves from daily records. Interna- tional Journal of Climatology, 37, 770–780. https://doi.org/10.1002/joc. 4738.

Rodríguez-Solá, R., Casas-Castillo, M.C., Navarro, X. and Redaño, A., 2017. A study of the scaling properties of rainfall in Spain and its appropriateness to generate intensity–duration–frequency curves from daily records. International Journal of Climatology, 37, 770–780. https://doi.org/10.1002/joc. 4738.

Rostami, R., 2013. Regional flood frequency analysis based on l-moment approach (case study: west Azarbaijan Basins). J. Civ. Eng. Urbanism (JCEU) 3 (3), 107–113.

Russell, B.T., 2019. Investigating precipitation extremes in South Carolina with focus on state's October 2015 precipitation event. Journal of Applied Statistics, 46 (2).

Satyanarayana, P., Srinivas, V. V., 2011. Regionalization of precipitation in data sparse areas using large scale atmospheric variables - a fuzzy clustering approach. J Hydrol 405:462–473.

Schertzer, D., Lovejoy, S., 2011. Multifractals, generalized scale invariance and complexity in geophysics. Int J Bifurcation Chaos 21(12):3417– 3456.

Schertzer, D., Lovejoy, S., 1987. Physical modelling and analysis of rain and clouds by anisotropic scaling multiplicative processes. Journal of Geophysical Research-Atmospheres 92: 9693–9714.

Svensson, C., Olsson, J., Berndtsson, R., 1996. Multifractal properties of daily rainfall in two different climates. Water Resources Research 32: 2463–2472.

Témez, J.R. 1987. Cálculo hidrometeorológico de caudales máximos en pequeñas cuencas naturales. Dirección general de carreteras. MOPU: Madrid.

Tessier, Y., Lovejoy, S., Hubert, P., Schertzer, D., Pecknold, S., 1996. Multifractal analysis and modelling of rainfall and river flows and scaling, causal transfer functions. Journal of Geophysical Research- Atmospheres 101: 26427–26440.

Valencia, J.L., Tarquis, A.M., Saa, A., Villeta, M., Gascó, J.M. 2015. Spatial Modeling of Rainfall Patterns over the Ebro River Basin Using Multifractality and

Non-Parametric Statistical Techniques. Water 7: 6204–6227. DOI:10.3390/w7116204

Veneziano, D., Furcolo, P., 2002. Multifractality of rainfall and scaling of intensity-duration-frequency curves. Water Resources Research 38(12). DOI: 10.1029/2001WR000372.

Veneziano, D., Furcolo, P., 2009. Improved moment scaling estimation for multifractal signals. Nonlin. Processes Geophys., 16, 641–653, 2009.

Xu, Y., Tung, Y., 2009. Constrained scaling approach for design rainfall estimation. Stoch Env Res RiskAssess 23:697–705.

Yang, T., Shao, Q., Hao, Z-C., Chen, X., Zhang, Z., Xu, C-Y., Sun, L., 2010. Regional Frequency Analysis and spatio-temporal pattern characterization of rainfall extremes in the Pearl River Basin. Journal of Hydrology 380(3–4): 386–405.

Yu, P. S., Yang, T. C., Lin, C. S., 2004. Regional rainfall intensity formulas based on scaling property of rainfall. J Hydrol 295(1–4): 108–123.

Yu, Z. G., Leung, Y., Chen, Y. D., Zhang, Q., Anh, V., Zhou, Y., 2014. Multifractal analyses of daily rainfall time series in Pearl River basin of China. Physica A, 405: 193-202.

Zakaria, Z. A., Shabri, A., 2012. Regional Frequency Analysis of extreme rainfalls using partial L-moments method. Theoretical and Applied Meteorology. DOI: 10.1007/s00704-012-0763-2.

Figure captions

Figure 1. Rain gauges at the Umbria Region (red triangles). The green squares show the rain gauges used in this work.

Figure 2. Log-log plot of the averaged q^{th} moments of the hourly rainfall intensity on the time scales from 1 hour to 21 days, versus the scale ratio λ at Casacastalda, San Benedetto Vecchio and San Silvestro stations. (a) Moments greater than 1; (b) Moments lower than 1. R^2 values higher than 0.9999 for all the fits.

Figure 3. Moments scaling exponent function $K(q)$ for the range of scales detected for the averaged q^{th} moments of the hourly rainfall intensity with the value of γ that makes $c(\gamma) = 1$ and the associated moment order q at Casacastalda, San Benedetto Vecchio and San Silvestro stations.

Figure 4. Values of the quantiles of 24 hours duration and 50 years of return period obtained by Local, Regional and In-site Regional Frequency Analysis for all the rain gauges used in this work.

Figure 5. (a) Comparison between the $Slopes_{IDF}$ (different color symbols for each frequency analysis) and the value of singularity γ_1, for Casacastalda, San Benedetto Vecchio and San Silvestro stations. (b) For each IDF studied (obtained from Local, Regional o In-site Regional Frequency Analyses quantiles), values of the mean rainfall intensity of all the durations analyzed and for different return periods, at Casacastalda, San Benedetto Vecchio and San Silvestro stations.

Figure A.1 Flowchart showing the application steps of the Regional Frequency Analysis.

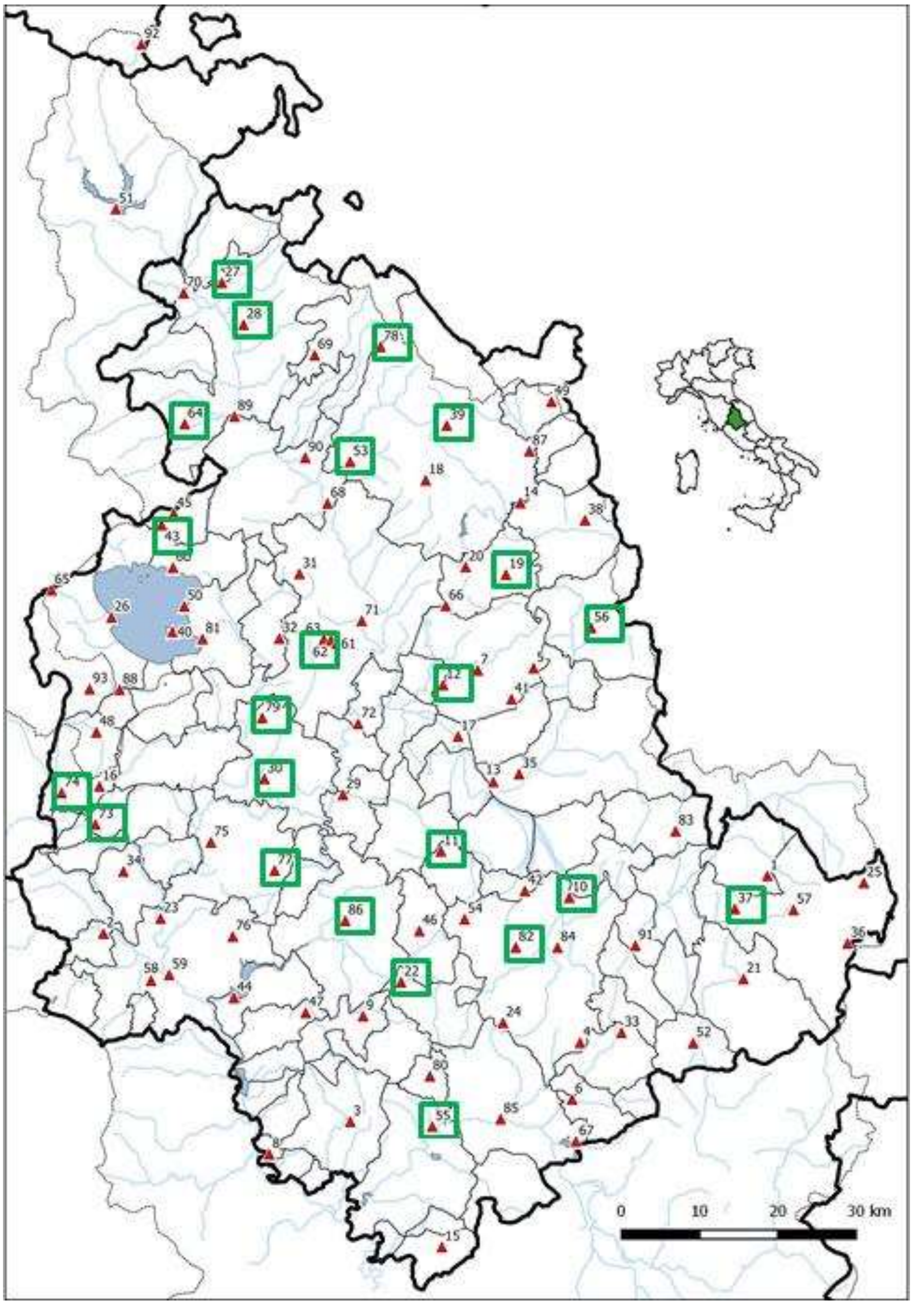

Figure 1. Rain gauges at the Umbria Region (red triangles). The green squares show the rain gauges used in this work

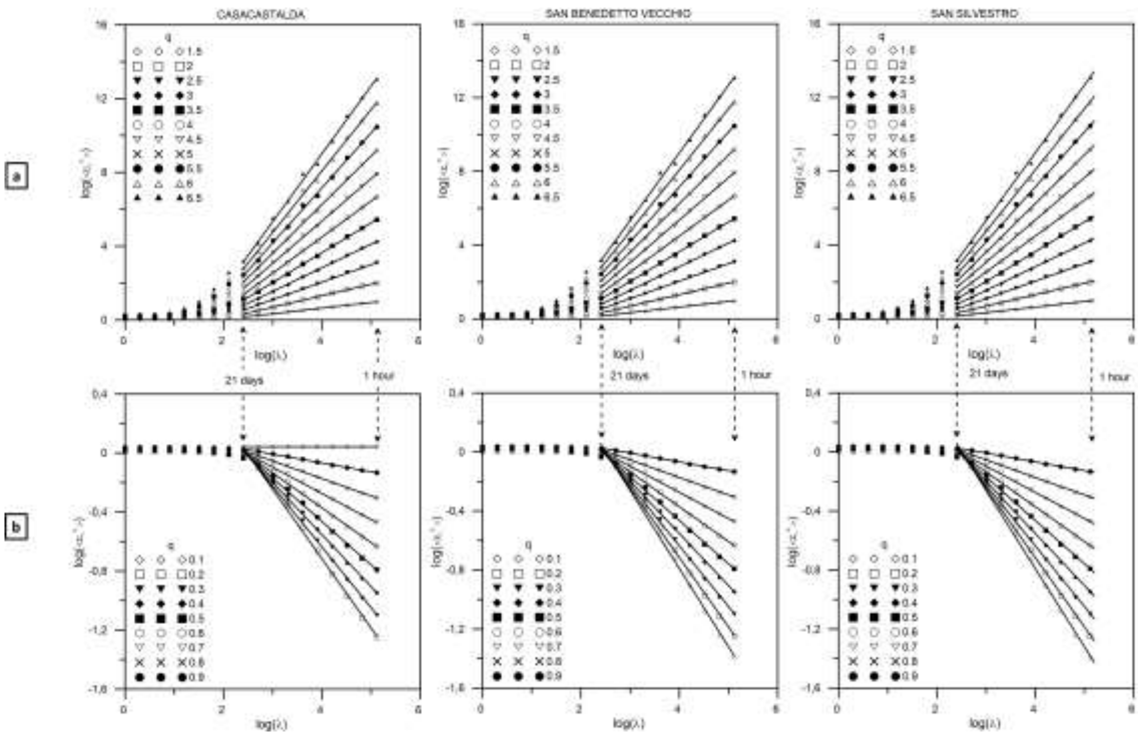

Figure 2. Log-log plot of the averaged q^{th} moments of the hourly rainfall intensity on the time scales from 1 hour to 21 days, versus the scale ratio λ at Casacastalda, San Benedetto Vecchio and San Silvestro stations. (a) Moments greater than 1; (b) Moments lower than 1. R^2 values higher than 0.9999 for all the fits.

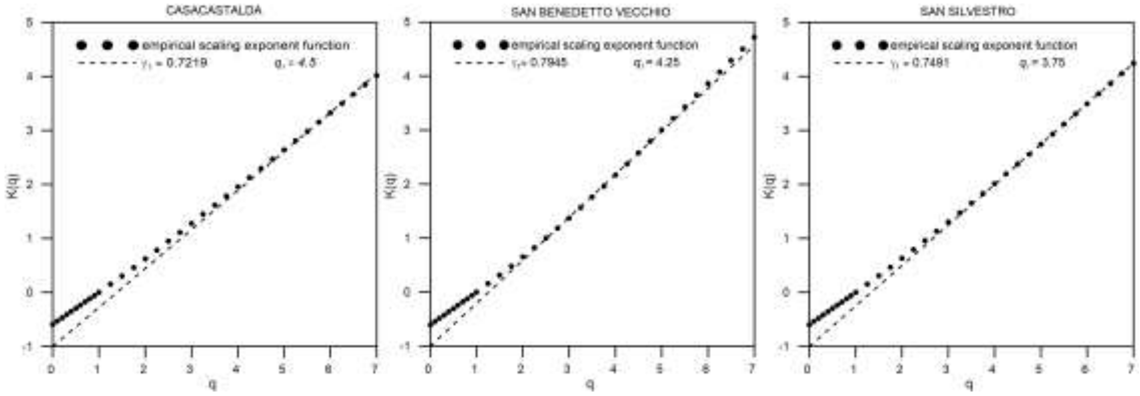

Figure 3. Moments scaling exponent function *K(q)* for the range of scales detected for the averaged q^{th} moments of the hourly rainfall intensity with the value of γ that makes $c(\gamma)$ = 1 and the associated moment order *q* at Casacastalda, San Benedetto Vecchio and San Silvestro stations.

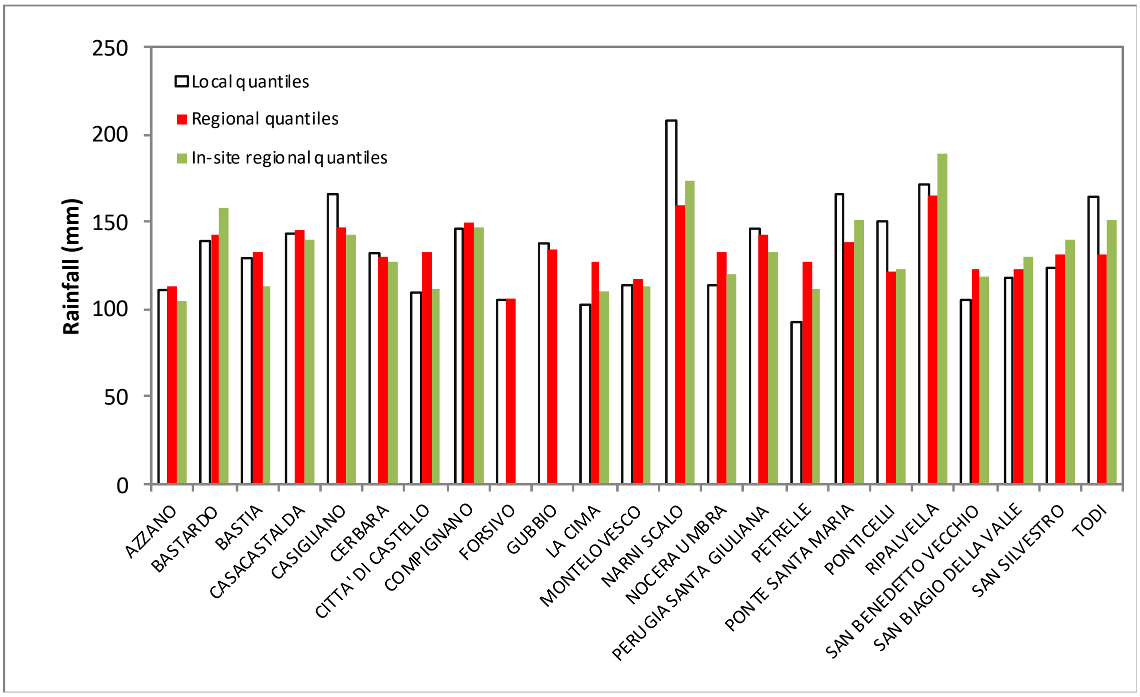

Figure 4. Values of the quantiles of 24 hours duration and 50 years of return period obtained by Local, Regional and In-site Regional Frequency Analysis for all the rain gauges used in this work.

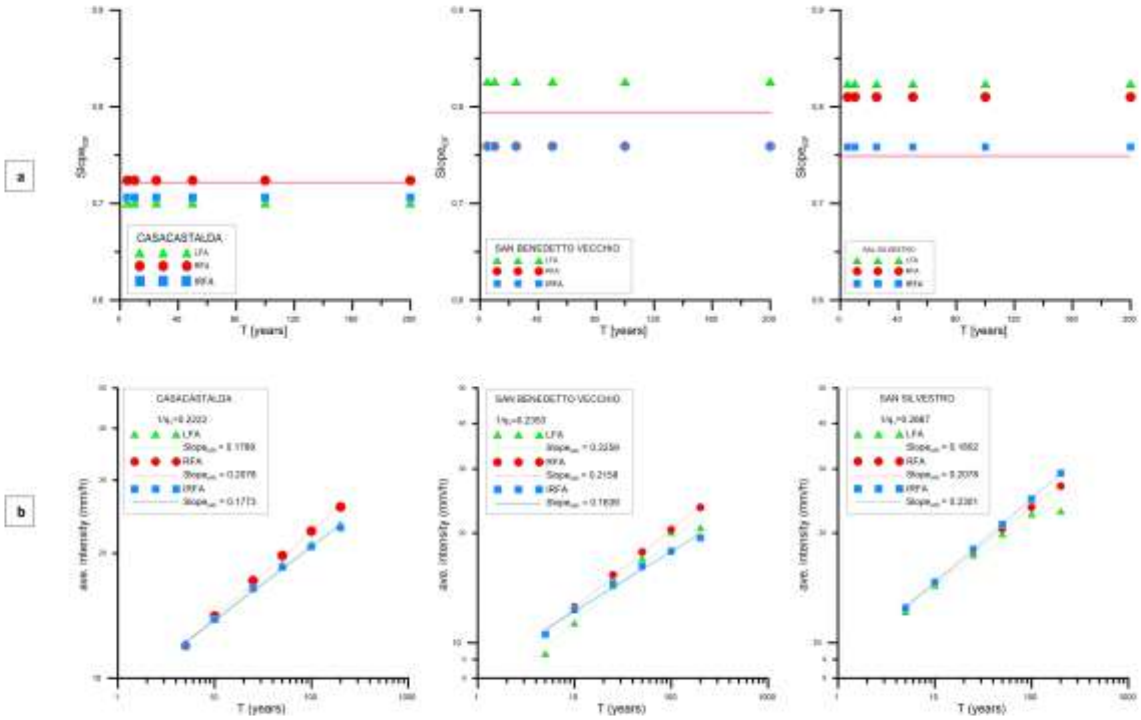

Figure 5. (a) Comparison between the $\mathrm{Slopes}_{\mathrm{IDF}}$ (different color symbols for each frequency analysis) and the value of singularity γ_1, for Casacastalda, San Benedetto Vecchio and San Silvestro stations. (b) For each IDF studied (obtained from Local, Regional o In-site Regional Frequency Analyses quantiles), values of the mean rainfall intensity of all the durations analyzed and for different return periods, at Casacastalda, San Benedetto Vecchio and San Silvestro stations.

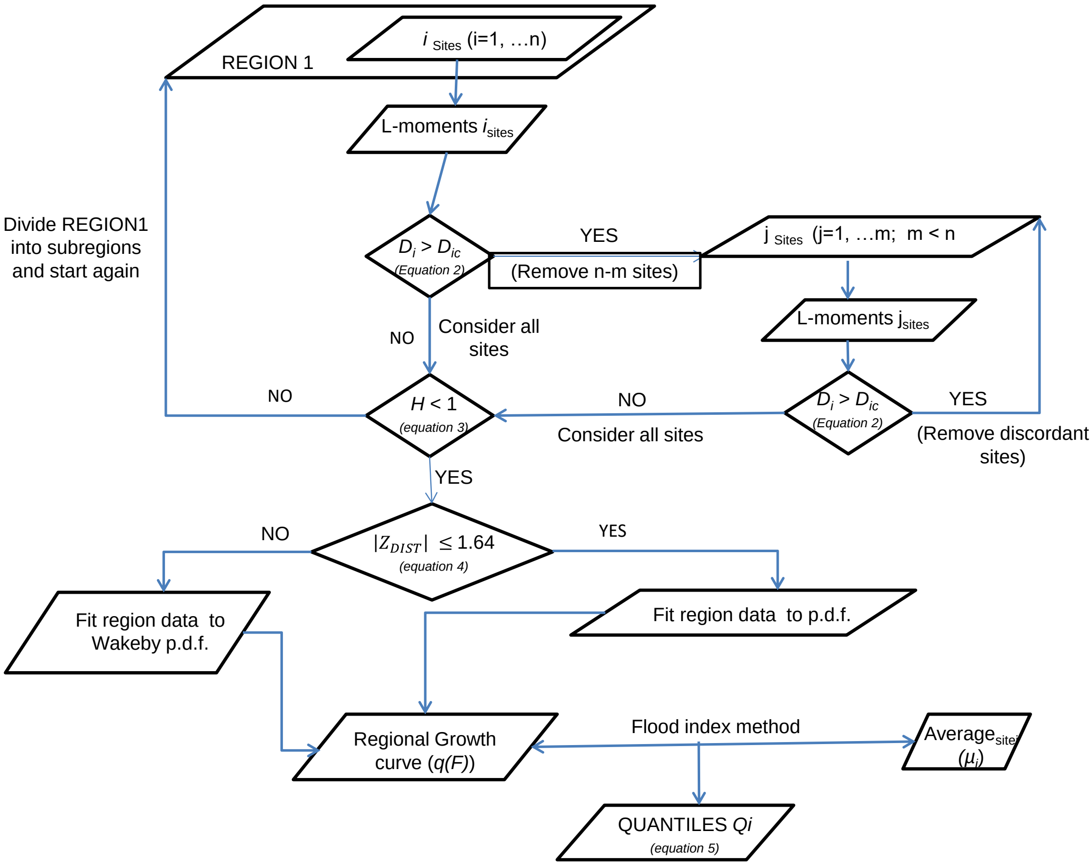

Figure A.1 Flowchart showing the application steps of the Regional Frequency Analysis.

Table captions

Table 1. Main characteristics of the rain gauges used in this work at the Umbria Region (Italy).

Table 2. Values of multifractal parameters obtained from the hourly rainfall data series analyzed. For symbols meaning see the text.

Table 3. Heterogeneity results of the Regional Frequency Analysis (RFA) performed for rainfall durations of 1h, 3h, 6h, 12h and 24h at the Umbria region. H is the heterogeneity measure.

Table 4. Values of statistics ZDIST for the five probability distribution functions tested at each homogenous region. In red color, the selected probability distribution function according to the value of ZDIST.

Table 5. Regional growth curves for different return periods (T) and durations obtained for the homogeneous regions by the selected probability distribution functions.

Table 6. In-site Regional Frequency Analysis results for all the stations at the Umbria region. H is the heterogeneity measure, pdf refers to most suitable probability distribution function for the region, and ZDIST is the statistics that measures the goodness of fit.

Table 7. In-site Regional Frequency Analysis growth curves for all the sites at the Umbria region and for different return periods (T), obtained from the selected probability distribution functions (details in table 6).

Table 8. Values of the IDF parameters, a and b, obtained by fitting the quantiles derived from Local, Regional and In-site Regional Frequency Analyses, for the stations of Casacastalda, San Benedetto Vecchio and San Silvestro.

Table 9. Absolute values of slopes of IDF curves slope, SlopeIDF, slopes and slope of average rainfall intensity fit versus return periods, SlopeARI, at each station obtained with the three adopted approaches (Local, Regional and In-site Regional Frequency Analyses).

Table 10. Comparison between the multifractal results and the IDF properties for the selection of the proper frequency analysis at each site. Coloured bold values select the best approach, being bold green for Local, bold red for Regional, and bold blue for In-site Regional Analysis.

Table 1. Main characteristics of the rain gauges used in this work at the Umbria Region (Italy).

RAIN GAUGE STATION	ID	ALTITUDE (m a.s.l.)	UTM33 X (m)	UTM33 Y (m)	MEAN ANNUAL RAINFALL (mm)
AZZANO	10	235	316615	4742431	782.5
BASTARDO	11	331	300489	4748742	803.8
BASTIA UMBRA	12	203	301377	4769716	705.0
CASACASTALDA	19	730	309715	4783398	971.0
CASIGLIANO	22	273	294947	4732331	869.1
CERBARA	27	310	275092	4821081	834.3
CITTÀ DI CASTELLO	28	304	277643	4815738	883.0
COMPIGNANO	30	240	278394	4758593	756.8
FORSIVO	37	963	337588	4740488	867.0
GUBBIO	39	471	302789	4802329	946.5
LA CIMA	43	791	266480	4790970	1097.1
MONTELOVESCO	53	634	290484	4798142	833.0
NARNI SCALO	55	109	298381	4713916	907.5
NOCERA UMBRA	56	534	320281	4776405	937.6
PERUGIA SANTA GIULIANA	62	417	287387	4775762	892.5
PETRELLE	64	342	269830	4803553	897.7
PONTE SANTA MARIA	73	240	256802	4753550	790.1
PONTICELLI	74	245	252657	4757685	754.0
RIPALVELLA	77	453	279329	4746964	879.1
SAN BENEDETTO VECCHIO	78	729	294749	4812427	831.5
SAN BIAGIO DELLA VALLE	79	257	278380	4766281	707.2
SAN SILVESTRO	82	381	309649	4736325	897.9
TODI	86	329	288089	4740319	852.0

Table 2. Values of multifractal parameters obtained from the hourly rainfall data series analyzed. For symbols meaning see the text.

STATION	γ_1	q_1	$K(0)$
AZZANO	0.8029	4.875	-0.6248
BASTARDO	0.7773	5.000	-0.6194
BASTIA UMBRA	0.8052	3.500	-0.6298
CASACASTALDA	0.7219	4.500	-0.5904
CASIGLIANO	0.7905	4.000	-0.6200
CERBARA	0.8052	3.500	-0.6342
CITTÀ DI CASTELLO	0.7406	5.875	-0.6050
COMPIGNANO	0.7881	4.750	-0.6579
FORSIVO	0.8219	4.000	-0.6395
GUBBIO	0.7973	3.750	-0.5932
LA CIMA	0.7838	4.500	-0.5895
MONTELOVESCO	0.7834	5.000	-0.6119
NARNI SCALO	0.7920	4.750	-0.6098
NOCERA UMBRA	0.7125	5.000	-0.5917
PERUGIA SANTA GIULIANA	0.7691	4.875	-0.6043
PETRELLE	0.7311	5.250	-0.6020
PONTE SANTA MARIA	0.8195	3.500	-0.6401
PONTICELLI	0.7659	5.000	-0.6241
RIPALVELLA	0.7759	4.000	-0.6036
SAN BENEDETTO VECCHIO	0.7945	4.250	-0.6091
SAN BIAGIO DELLA VALLE	0.8242	3.500	-0.6389
SAN SILVESTRO	0.7491	3.750	-0.5986
TODI	0.8056	3.500	-0.5899

Table 3. Heterogeneity results of the Regional Frequency Analysis (RFA) performed for rainfall durations of 1h, 3h, 6h, 12h and 24h at the Umbria region. H is the heterogeneity measure.

RFA_{ih}	Number of stations	Considered stations (ID)	*H*
RFA_{1h}	22	All except 64	-0.11
RFA_{3h}	21	All except 30 and 79	0.90
RFA_{6h}	23	-	1.96
RFA_{12h}	21	All except 55 and 78	0.54
RFA_{24h}	23	-	0.97
RFA_{6hA}	8	19, 27, 53, 56, 62, 74, 79, 82	-0.23
RFA_{6hB}	14	10, 11, 12, 22, 28, 30, 37, 39, 43, 55, 73, 77, 78, 86	-0.83

Table 4. Values of statistics ZDIST for the five probability distribution functions tested at each homogenous region. In red color, the selected probability distribution function according to the value of ZDIST.

RFA_{ih}	GEN-LOG	GEV	GEN-NOR	PT-III	GEN-PAR
RFA_{1h}	0.87	-1.04	-1.56	-2.60	-5.45
RFA_{3h}	1.59	-0.25	-0.91	-2.15	-4.63
RFA_{6hA}	1.47	0.03	-0.11	-0.54	-3.08
RFA_{6hB}	2.25	0.76	0.13	-1.01	-2.87
RFA_{12h}	2.54	0.43	-0.02	-0.98	-4.36
RFA_{24h}	1.81	-0.31	-0.81	-1.84	-5.17

GEN-LOG: generalized logistic; GEV: generalized extreme value; GEN-NOR: generalized normal; PT-III: Pearson Type

Table 5. Regional growth curves for different return periods (T) and durations obtained for the homogeneous regions by the selected probability distribution functions.

	T (years)					
RFA_{ih}	5	10	25	50	100	200
RFA_{1h}	1.23	1.46	1.79	2.07	2.40	2.79
RFA_{3h}	1.24	1.46	1.78	2.02	2.29	2.57
RFA_{6hA}	1.22	1.40	1.63	1.80	1.97	2.14
RFA_{6hB}	1.25	1.49	1.81	2.06	2.33	2.60
RFA_{12h}	1.24	1.45	1.72	1.92	2.12	2.32
RFA_{24h}	1.25	1.47	1.76	1.98	2.21	2.45

Table 6. In-site Regional Frequency Analysis results for all the stations at the Umbria region. H is the heterogeneity measure, pdf refers to most suitable probability distribution function for the region, and ZDIST is the statistics that measures the goodness of fit.

In-site Region		**Considered sites**	***H***	**pdf**	Z^{DIST}
AZZANO		All	-2.08	PT-III	0.62
BASTARDO		All	-1.47	GEV	0.08
BASTIA UMBRA		All	-0.76	GEN-PAR	-1.40
CASACASTALDA		All	-1.14	GEV	0.08
CASIGLIANO		All	-1.23	PT-III	-0.34
CERBARA		All	-1.79	GEN-NOR	-0.29
CITTÀ DI CASTELLO		All	-1.87	PT-III	-0.26
COMPIGNANO		All	-2.59	GEN-NOR	0.16
FORSIVO		All except 5’, 24h	-1.04	GEN LOG	0.23
GUBBIO		All except 24 h	0.99	GEN-NOR	0.19
LA CIMA		All except 5’, 10’, 15’, 20’	-0.93	GEN-NOR	-0.02
MONTELOVESCO	1	5’, 10’, 15’, 20’, 30’, 40’	-0.33	GEN-PAR	-1.17
	2	1, 3, 6, 12, 24 h	0.00	PT-III	0.99
NARNI SCALO		30’, 1, 3, 6, 12, 24 h	0.85	PT-III	0.85
NOCERA UMBRA		All	-1.52	PT-III	0.01
PERUGIA SANTA GIULIANA		All	-1.94	GEN-LOG	0.19
PETRELLE	1	5’, 10’, 15’, 20’, 12h	0.88	PT-III	1.45
	2	30’, 40’, 1, 3, 24 h	-0.38	GEN-PAR	1.17
PONTE SANTA MARIA		All	-0.43	GEN-NOR	-0.05
PONTICELLI		All	0.23	PT-III	0.07
RIPALVELLA		All	-1.29	GEN-NOR	0.17
SAN BENEDETTO VECCHIO		30’, 1, 3, 6, 12, 24 h	0.99	PT-III	-0.17
SAN BIAGIO DELLA VALLE		30’, 40’, 1, 3, 6, 12, 24 h	-1.89	GEN-LOG	-1.21
SAN SILVESTRO		All	-1.77	GEN-LOG	-0.11
TODI		All	0.88	GEV	-0.42

Table 7. In-site Regional Frequency Analysis growth curves for all the sites at the Umbria region and for different return periods (T), obtained from the selected probability distribution functions (details in table 6).

	***T* (years)**					
In Site Region	**5**	**10**	**25**	**50**	**100**	**200**
AZZANO	1.250	1.447	1.584	1.854	2.018	2.177
BASTARDO	1.227	1.475	1.836	2.142	2.482	2.862
BASTIA UMBRA	1.283	1.465	1.635	1.725	1.791	1.840
CASACASTALDA	1.196	1.385	1.644	1.852	2.072	2.306
CASIGLIANO	1.246	1.465	1.739	1.938	2.134	2.325
CERBARA	1.259	1.441	1.658	1.813	1.962	2.107
CITTÀ DI CASTELLO	1.222	1.396	1.607	1.757	1.903	2.044
COMPIGNANO	1.265	1.479	1.747	1.946	2.144	2.344
FORSIVO	1.264	1.498	1.830	2.112	2.427	2.782
GUBBIO	1.246	1.480	1.792	2.036	2.288	2.551
LA CIMA	1.253	1.413	1.596	1.722	1.840	1.953
MONTELOVESCO1	1.316	1.449	1.543	1.580	1.601	1.613
MONTELOVESCO2	1.245	1.425	1.639	1.790	1.934	2.074
NARNI SCALO	1.305	1.613	1.993	2.262	2.515	2.753
NOCERA UMBRA	1.225	1.383	1.568	1.697	1.820	1.937
PERUGIA SANTA GIULIANA	1.213	1.383	1.617	1.807	2.014	2.240
PETRELLE1	1.203	1.345	1.512	1.629	1.740	1.846
PETRELLE2	1.302	1.480	1.638	1.717	1.772	1.811
PONTE SANTA MARIA	1.291	1.542	1.865	2.109	2.357	2.609
PONTICELLI	1.276	1.482	1.729	1.904	2.072	2.234
RIPALVELLA	1.303	1.596	1.988	2.295	2.614	2.947
SAN BENEDETTO VECCHIO	1.232	1.441	1.703	1.894	2.081	2.266
SAN BIAGIO DELLA VALLE	1.213	1.417	1.715	1.974	2.271	2.614
SAN SILVESTRO	1.195	1.408	1.733	2.030	2.382	2.802
TODI	1.234	1.499	1.890	2.228	2.608	3.039

Table 8. Values of the IDF parameters, a and b, obtained by fitting the quantiles derived from Local, Regional and In-site Regional Frequency Analyses, for the stations of Casacastalda, San Benedetto Vecchio and San Silvestro.

STATION	**T (years)**	**IDF-LOCAL**		**IDF-RFA**		**IDF-IRFA**	
		a	***b***	***a***	***b***	***a***	***b***
CASACASTALDA	5	35.54	0.2992	36.55	0.2760	35.53	0.2938
	10	41.19		43.06		41.14	
	25	48.95		52.37		48.83	
	50	55.20		60.25		54.99	
	100	61.84		69.05		61.54	
	200	68.93		78.95		68.50	
SAN BENEDETTO VECCHIO	5	33.40	0.1734	34.05	0.2409	33.96	0.2414
	10	40.52		40.37		39.72	
	25	51.33		49.48		46.94	
	50	60.88		57.23		52.21	
	100	71.89		65.92		57.37	
	200	84.61		75.75		62.46	
SAN SILVESTRO	5	43.34	0.1759	43.50	0.1898	42.25	0.2076
	10	51.22		51.24		49.75	
	25	62.12		62.32		61.27	
	50	70.93		71.70		71.74	
	100	80.36		82.18		84.18	
	200	90.47		93.94		99.06	

Table 9. Absolute values of slopes of IDF curves slope, SlopeIDF, slopes and slope of average rainfall intensity fit versus return periods, SlopeARI, at each station obtained with the three adopted approaches (Local, Regional and In-site Regional Frequency Analyses).

STATION	Slope$_{IDF}$			Slope$_{ARI}$		
	LFA	RFA	IRFA	LFA	RFA	IRFA
AZZANO	0.7492	0.7892	0.7730	0.1543	0.2243	0.1489
BASTARDO	0.7999	0.7673	0.7510	0.2563	0.2161	0.2288
BASTIA UMBRA	0.7276	0.7463	0.7300	0.1829	0.2161	0.0952
CASACASTALDA	0.7007	0.7240	0.7062	0.1789	0.2078	0.1773
CASIGLIANO	0.7046	0.7623	0.7460	0.1795	0.2161	0.1677
CERBARA	0.7363	0.7563	0.7385	0.1819	0.2078	0.1383
CITTÀ DI CASTELLO	0.7095	0.6913	0.7045	0.1590	0.2161	0.1384
COMPIGNANO	0.7272	0.7620	0.7414	0.1336	0.2186	0.1657
FORSIVO	0.8646	0.8373	0.8156	0.2257	0.2200	0.2126
GUBBIO	0.7418	0.7264	0.7030	0.0434	0.2161	0.1932
LA CIMA	0.8111	0.7713	0.8233	0.1398	0.2268	0.1190
MONTELOVESCO	0.7205	0.7697	0.7501	0.1091	0.2077	0.1453
NARNI SCALO	0.6920	0.7556	0.7248	0.2213	0.2158	0.1999
NOCERA UMBRA	0.7265	0.7175	0.6997	0.1320	0.2078	0.1231
PERUGIA SANTA GIULIANA	0.7156	0.7466	0.7288	0.1497	0.2077	0.1655
PETRELLE	0.8239	0.7262	0.7276	0.1534	0.2026	0.0860
PONTE SANTA MARIA	0.7215	0.7469	0.7306	0.2041	0.2161	0.1712
PONTICELLI	0.6425	0.7431	0.7252	0.1280	0.2078	0.1502
RIPALVELLA	0.7395	0.6879	0.6716	0.2498	0.2161	0.2196
SAN BENEDETTO VECCHIO	0.8263	0.7591	0.7586	0.2259	0.2158	0.1639
SAN BIAGIO DELLA VALLE	0.7638	0.7461	0.7139	0.1820	0.2057	0.2072
SAN SILVESTRO	0.8241	0.8102	0.7924	0.1802	0.2078	0.2301
TODI	0.7444	0.7662	0.7499	0.2517	0.2161	0.2434
AZZANO	0.7492	0.7892	0.7730	0.1543	0.2243	0.1489
BASTARDO	0.7999	0.7673	0.7510	0.2563	0.2161	0.2288

Table 10. Comparison between the multifractal results and the IDF properties for the selection of the proper frequency analysis at each site. Coloured bold values select the best approach, being bold green for Local, bold red for Regional, and bold blue for In-site Regional Analysis.

STATION	$(\gamma_1 - Slope_{IDF})^2$			$\left\|Slope_{ARI} - (1/q_{1})\right\|$		
	LFA	RFA	IRFA	LFA	RFA	IRFA
AZZANO	0.00288	**0.00019**	0.00089	0.0508	**0.0192**	0.0562
BASTARDO	0.00051	**0.00010**	0.00069	0.0563	**0.0161**	0.0288
BASTIA UMBRA	0.00602	**0.00347**	0.00566	0.1028	**0.0696**	0.1905
CASACASTALDA	0.00045	**0.00000**	0.00025	0.0433	**0.0144**	0.0449
CASIGLIANO	0.00738	**0.00080**	0.00198	0.0705	**0.0339**	0.0823
CERBARA	0.00475	**0.00239**	0.00445	0.1038	**0.0779**	0.1474
CITTÀ DI CASTELLO	**0.00097**	0.00243	0.00130	**0.0112**	0.0459	0.0318
COMPIGNANO	0.00371	**0.00068**	0.00218	0.0769	**0.0081**	0.0448
FORSIVO	0.00182	0.00024	0.00004	0.0243	0.0300	0.0374
GUBBIO	0.00308	0.00503	0.00889	0.2233	0.0506	0.0735
LA CIMA	0.00075	**0.00016**	0.00156	0.0824	**0.0046**	0.1032
MONTELOVESCO	0.00396	**0.00019**	0.00111	0.0909	**0.0077**	0.0547
NARNI SCALO	0.01000	**0.00132**	0.00452	0.0108	**0.0053**	0.0106
NOCERA UMBRA	0.00020	**0.00003**	0.00016	0.0680	**0.0078**	0.0769
PERUGIA SANTA GIULIANA	0.00286	**0.00051**	0.00162	0.0554	**0.0026**	0.0396
PETRELLE	0.00861	0.00002	0.00001	0.0371	0.0121	0.1045
PONTE SANTA MARIA	0.00960	**0.00527**	0.00790	0.0816	**0.0696**	0.1145
PONTICELLI	0.01523	**0.00052**	0.00166	0.0720	**0.0078**	0.0498
RIPALVELLA	**0.00132**	0.00774	0.01088	**0.0002**	0.0339	0.0304
SAN BENEDETTO VECCHIO	**0.00101**	0.00125	0.00129	**0.0094**	0.0195	0.0714
SAN BIAGIO DELLA VALLE	0.00365	0.00610	0.01217	0.1037	0.0800	0.0785
SAN SILVESTRO	0.00563	0.00373	**0.00187**	0.0865	0.0589	**0.0366**
TODI	0.00375	0.00155	0.00310	0.0340	0.0696	0.0423