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Influence of spatial correlation of core strength measurements on the assessment of in-situ concrete strength

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Abstract

Many countries are experiencing an increasing need of checking the safety of existing structures. The assessment of structural capacity of RC structures strictly depends on the in-situ compressive strength of concrete. The evaluation of this property is typically carried out by means of destructive tests on concrete cores taken from the structure. The experimental data is then interpreted using a relevant code to obtain a design strength value according to a required percentile and confidence. In this paper the principal international standards that deal with the statistical interpretation of data from concrete core test are presented. Since it is reasonable to assume that concrete strength is a realization of a random field, the assumption of statistical independence of core test data is questioned. An extension of the classical theory of tolerance limits in the case of normally distributed correlated samples is thus proposed. Finally, application examples of this methodology are

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provided to illustrate some important implications of the spatial correlation of core test values on concrete strength estimations.

Keywords: Existing Structures, In-situ Concrete Strength, Core Testing, Spatial Correlation, Tolerance Limits

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1. Introduction

In the last decades many countries have experienced an increasing need of assessing the performances of old buildings and infrastructures. The evaluation of existing structures is becoming a prominent priority in many countries where strong earthquakes are frequent and where a great share of the built heritage dates back to just after the World War II, when no code prescriptions were available to protect the structures against the seismic action. An accurate evaluation of the existing structures may allow to plan and execute strengthening interventions to reduce causalities in case of earthquakes and to guarantee the functionality of strategic structures, such as hospitals, when such extreme events occur.

The need of assessing existing structural systems may also be due to their aging and degradation. As an example, in the United States the various Departments of Transportation have the duty of periodically checking the conditions of existing bridges. In case of necessity an evaluation of the residual load bearing capacity has to be performed either to post the bridge for load or to plan and execute a repair or strengthening intervention. The most recent data contained in the National Bridge Inventory Database suggest that about 10% of existing bridges in the United States are structurally deficient. It is thus clear how the evaluation of the safety of existing bridges is a task that is as important and critical as guaranteeing the safety of new ones.

Finally, in countries were an old built heritage is available, the need of both preserving and reusing the traditional constructions leads to the necessity of assessing the structural capacity against new load conditions.

The result of all these different needs is that several countries have developed codes specifically aimed at providing tools and guidelines for the assessment of existing structures. As an example, in Europe prescriptions for performing these kind of evaluations have been given in the Eurocode 8 [1], specifically to address the problem of checking the safety of old buildings against earthquake-induced actions. In the United States, the American Concrete Institute released the ACI 562-16 [2] with the intention of providing minimal guidelines for the evaluation, repair and strengthening of existing RC structures. Similar indications have been released by AASHTO with the Manual for Bridge Evaluation [3] to give instructions specifically aimed at evaluating and rating the structural conditions of existing bridges.

Any kind of in-depth structural evaluation must take into consideration the properties of structural materials. Compressive strength of concrete is surely one of the prominent factors which affects the overall safety of a RC structure. Any assessment begins with a survey of the structural system and of the existing documentation, which may contain information also on the materials that had been used for the construction. However oftentimes these documents have been lost or may be unreliable, so that an experimental evaluation of the material properties is almost always required.

The assessment of in-situ concrete compressive strength is typically performed extracting concrete cores from the structure and then by testing them in compression testing machines. This type of evaluation can be integrated by the use of non-destructive techniques, like SonReb tests, which however always require a proper calibration with destructive data to provide meaningful information and are particularly sensitive to external factors such as concrete carbonation [4, 5] and water content.

Many uncertainties are involved in the evaluation of in-situ concrete core strength results and thus statistical tools are needed to interpret test data. This need is even more relevant for old RC structures and infrastructures built before the eighties, for which is known that the quality of the material and workmanship were far below the actual practice [6].

Several standards have been published to give details on how a correct assessment of in-situ strength of concrete should be performed. Nevertheless, these codes adopt different ways of interpreting the core strength test results, and some of them are even scarcely justified. Furthermore all existing standards implicitly assume that the measured core strength values are independent one to each other, even though it is reasonable to think that in-situ concrete strength is actually a realization of a random field with a certain correlation function.

One of the consequences of the assumption of independence of sample test values is that codes e.g. ACI 214.4R [7] suggest to choose core locations at random. In presence of spatial correlation however more rational sampling schemes should be developed to optimally extract cores so to maximize the amount of information on the field. With regard to this problem, recent researches [8, 9] are promoting the use of NDT data to select in a more rational and representative way the sampling locations, rather than haphazardly choosing them.

This work is thus aimed at providing a consistent statistical framework, within the statistical theory of tolerance limits, to investigate the influence of spatial correlation of core test values on the confidence of in-situ concrete strength assessment. In detail the objective has been the generalization of the tolerance factor method of the ACI 214.4R code to make it applicable to any correlation function. The advantage of this latter compared to other literature approaches currently in use is that it is statistically well-supported and tunable, as the user can select the desired confidence level in the estimates.

This basic framework might be used for the definition of more accurate assessment procedures which are able to take into account the levels of correlation of the material strength measurements. The proposed approach does not consider epistemic uncertainties, which may turn out to be not-negligible [10], but focuses its efforts in the reduction of the effects of aleatory uncertainty in the estimates due to the spatial correlation of strength measurements.

2. Current approaches

In this section the most relevant standards that deal with the assessment of in-situ strength of concrete using cores are presented. The ACI 562-14 [2] is one of the most widely recognized codes for the assessment of existing structures. Its prescriptions on the evaluation of core test results are directly derived from the ACI 214-4R [7] that will be presented in the following section. For what it concerns European standards, the EN 13791:2007 is the main document that deals with the assessment of in-situ concretes, even though Eurocode 8 [1] gives different prescriptions for what concerns the seismic evaluation of existing structures.

2.1. ACI 214-4R

The ACI 214-4R is a set of guidelines for the extraction of cores and interpretation of the compression test results. This document suggests two different approaches for the estimation of an equivalent in-situ strength value to be used for the evaluation of the structural capacity of an existing structure.

Given a set of n core test data x_i with $i = 1 \dots n$, the ACI 214-4R suggests to correct these values to account for their different testing conditions (core diameter, length to diameter ratio, moisture content, damage due to drilling, etc.) multiplying the results by strength correction factors provided by the code itself. Since these factors have been empirically obtained by statistical interpretation of experimental results, they are subjected to a certain statistical variability [11], which must be accounted for in performing the evaluations. This is accomplished taking into consideration the standard deviation s_a of these correction coefficients, which is given by the code itself.

After having homogenized the test values, the interpretation of the results can be carried out following two approaches. The first one is termed the *tolerance factor method*, as it is based on the statistical theory of tolerance regions. Following this approach the estimation $f_{p,est}$ of a given p^{th} percentile f_p of the in-situ concrete strength distribution with a desired confidence level is obtained as:

$$f_{p,est} = \bar{x}_s - k s_s \tag{1}$$

where $\bar{x}_s$ is the mean value of the corrected test results, s_s is their standard deviation and k is a coefficient that depends on the desired confidence $1 - \alpha$ (where α is the chosen probability of overestimating the given percentile) and on the number of samples n, under the hypotheses of independent and normally distributed samples. This coefficient can be evaluated [12] as:

$$k = k(n, p, \alpha) = t^{-1}_{n-1,1-\alpha}(z_{1-p}\sqrt{n})/\sqrt{n} \tag{2}$$

where $t^{-1}_{n-1,1-\alpha}(x)$ is the inverse non-central t distribution with $n-1$ degrees of freedom evaluated in $1-\alpha$ and with non-centrality parameter x. The term z_x represents the inverse cumulative distribution function of a standard normal distribution evaluated in x.

If the variability due to the uncertainty in the strength correction factors is accounted for, the following expression should be used:

$$f_{p,est} = \bar{x}_s - \sqrt{(ks_s)^2 + (Zs_a)^2} \tag{3}$$

where Z is a coefficient provided in the code as a function of the desired confidence level.

Alongside this approach, the ACI 214.4R defines an *alternate method* that is mainly based on the research of Bartlett and MacGregor [13]. These Authors stated that the tolerance factor approach may be too conservative mostly for two reasons. First of all, in their opinion the measured core test values overestimate the actual variability of the in-place concrete strength [14], furthermore they believe that this approach is too precise for the requirements of actual design practice.

As a consequence, the alternate method is less conservative. It is aimed at estimating the 10% percentile of concrete strength and it consists in a two-step approach. First a 90% lower confidence limit for the mean $\bar{x}_{s,90}$ is estimated using an equation that is formally analogous to Eq.(3). Then this value is reduced to obtain the equivalent specified strength as:

$$f_{c,eq} = \bar{x}_{s,90}(1 - 1.28V_{ws}) \tag{4}$$

where V_{ws} is the within-structure coefficient of variation of concrete strength given by the code itself. This latter value has been experimentally obtained by interpreting literature data. This approach uses core test data only to obtain an estimation of the average in-place strength, whereas the additional variation of concrete strength within the structure is modeled using experimentally based literature values. The result is that the alternate method yields values that in general are significantly higher than those achieved by the tolerance factor approach.

2.2. EN 13791:2007

The European Standard EN 13791:2007 [15] gives prescriptions on the assessment of in-situ compressive strength in structures and in precast concrete elements either by extraction and testing of concrete cores or by the use of indirect non-destructive methods.

The assessment of the characteristic (i.e. relative to a 5% percentile) in-situ concrete strength by core testing is carried out following two different approaches depending on the number of available cores. Approach A must be used when at least 15 cores are available. In this case the in-situ characteristic strength estimation $f_{ck,est}$ is given by:

$$f_{ck,est} = min\left[\bar{x}_s - min(2, s_s) \cdot k_2, x_{min} + 4\right] \tag{5}$$

where k_2 is a coefficient typically assumed to be equal to 1.48, x_{min} is the minimum of the measured core strength values. In the previous equation all the values should be expressed in MPa.

If 3 to 14 cores have been tested, then approach B is required. In this case the estimation is given by:

$$f_{ck,est} = min\left(\bar{x}_s - k, x_{min} + 4\right) \tag{6}$$

where k depends on the number n of cores.

The criteria adopted by the EN 13791:2007 are clearly derived from the ones suggested in the European Standard EN 206 [16] for checking the compliance of concrete production. The purpose and hypotheses of this latter code are however different from that of the EN 13791. The philosophy behind the EN 206 is aimed at addressing and balancing two opposing interests: the consumer's risk (erroneous acceptance of a non-compliant concrete lot) and the concrete producer's risk (erroneous rejection of a conforming production). More details on the criteria of EN 206 can be found in the works of Taerwe [17] [18]. However, in the case of the assessment of in-situ concrete strength the objective and boundary conditions are completely different. The aim is no more that of checking the compliance of concrete production to a given design requirement, but simply to assess an existing material property. The figure of the producer doesn't exists anymore and as such it is not justifiable to put the basis of the evaluation on a theory that tries not to unduly penalize the producer.

2.3. Eurocode 8

The EN 1998, also named Eurocode 8, is that part of the European design codes that deals with the seismic design of structures. The part 3 of Eurocode 8 [1] gives prescriptions for the evaluation of existing buildings subject to seismic actions. According to these guidelines, the evaluation of structural material properties should be obtained using both original design data, if available, and experimental tests data.

The design strength value to be used in the structural analysis is simply given by the average value of test results divided by a *confidence factor* (CF) which depends on the knowledge level (KL) of the structure.

$$f_{c,est} = \frac{\bar{x}_s}{CF_{KL}} \tag{7}$$

However, the knowledge level depends not only on the amount of data relative to material properties, but also on the available information on the structural geometry and its detailing (size and layout of steel reinforcement). As a result the confidence factors are rather arbitrary and they range between 1.35 (for the worst knowledge level KL1) and 1.00 (for the best knowledge level KL3).

It is clear that this approach is mostly empirical, and it is questionable to assume, within a semi-probabilistic structural design framework, that the uncertainties on the knowledge of structural materials are taken into account using such a scarcely justified approach. Additionally it is not realistic to assume the absence of uncertainties (i.e. CF=1.00) in the case of KL3, since the state of perfect knowledge is practically unattainable. Research efforts aimed at benchmarking the appropriateness of the Eurocode confidence factors for specific case studies can be found in literature [19].

3. Generalization of tolerance factor method to the case of correlated samples

In this section a generalization of the tolerance limit theory to the case of correlated samples is presented.

3.1. Fundamentals of tolerance limits

The theory of tolerance limits is a subset of the more general topic of tolerance regions, which has been treated in depth, among others, by Guttman [20]. In very basic terms, if n random variables $\mathbf{x}_i$, $i = 1 \dots n$ are defined so that $\mathbb{R}$ is their sample space and U their associated σ-Algebra, then a *tolerance region* is a statistic that maps the random point $(\mathbf{x}_1, \dots, \mathbf{x}_n)$ into a region $S(\mathbf{x}_1, \dots, \mathbf{x}_n)$ defined in U. The boldface notation from now on will be used to denote random quantities, whereas the uppercase notation is used for vectors and matrices.

If the region is defined as:

$$S(\mathbf{x}_1, \dots, \mathbf{x}_n) = [L_1(\mathbf{x}_1, \dots, \mathbf{x}_n), L_2(\mathbf{x}_1, \dots, \mathbf{x}_n)] \tag{8}$$

then S is defined a *tolerance interval*.

If $L_1(\cdot)$ is set to be equal to $-\infty$ then the tolerance interval becomes $S(\mathbf{x}_1, \dots, \mathbf{x}_n) = [-\infty, L_2(\mathbf{x}_1, \dots, \mathbf{x}_n)]$. In the context of the estimation of a given p-percentile of concrete strength the random variables $\mathbf{x}_1, \dots, \mathbf{x}_n$ represent the random outcomes of the test on n concrete cores. What it is desirable is to obtain a value for L_2 such that no more than a fraction p of the population of strengths is greater than L_2 in $1 - \alpha$ percent of the cases.

Using a mathematical notation similar to that presented by Guttman [20], such a condition can be described as:

$$Pr\left\{Pr_{\mathbf{x}}[\mathbf{x} \leq L_2(\mathbf{x}_1, \ldots, \mathbf{x}_n)] \leq p\right\} = 1 - \alpha \tag{9}$$

The inner probability operator represents the probability that a generic random variable $\mathbf{x}$ with the same distribution of the sample values but independent from these latter is lower than L_2. Since $L_2(\cdot)$ is a random function of the sample values, the inner probability is itself a random variable. If the stated condition holds, then $L_2(\cdot)$ is called *lower tolerance limit*.

The problem is thus to derive an expression for the lower tolerance limit $L_2(\cdot)$. Traditionally this has always been expressed by an equation of the form:

$$L_2(\alpha, p, n) = \bar{\mathbf{x}}_s - k(\alpha, p, n)\mathbf{s}_s \tag{10}$$

where $\bar{\mathbf{x}}_s$ is the sample average and $\mathbf{s}_s$ is the sample standard deviation. In the case of normally distributed samples Equation (9) can be rewritten, together with Equation (10), in a simpler form by dropping the inner probability operator:

$$Pr\left\{\bar{\mathbf{x}}_s - k(\alpha, p, n)\mathbf{s}_s \leq f_p\right\} = 1 - \alpha \tag{11}$$

where f_p is the p^{th} percentile that should be estimated and $\bar{\mathbf{x}}_s - k(\alpha, p, n)\mathbf{s}_s$ is the lower tolerance limit estimator.

The problem now is to find a proper expression for k so that Equation (11) holds. It is well-known that the exact solution to this problem in the case of independent samples is represented by Equation (2). Once k is known, an estimation of the p^{th} percentile with the desired confidence $1 - \alpha$ can thus be obtained by applying Equation (10), which is the same of Equation (1) given in the ACI 214.4R standard.

Even if it can be questioned that a normal distribution is not the best choice to represent non negative quantities such as concrete compressive strength, this assumption is well-estabilished in literature and recent investigations [21] confirmed that its use is acceptable to describe the in-situ concrete strength variability. Conversely, the assumption of independence of samples upon which Equation (2) relies is questionable. In the following this hypothesis will be removed to derive a generalization to the case of a known correlation law.

3.2. Assumptions

In the analyses it will be assumed that concrete strength is distributed as an homogeneous Gaussian random field with known correlation law.

The assumption of Gaussianity, while representing an approximation as concrete compressive strength cannot assume negative values, is typically accepted both in the setting of concrete testing, as previously stated, and in the field of reliability analyses of concrete structures (see e.g. [22, 23, 24, 25, 26]) and as such has been used in the developement of the current work. However if the use of lognormal fields is deemed to be necessary, the resulting tools will still be applicable even to this type of fields by a proper mapping of the data points to the associated normal field and assuming that the correlation function of this latter is known. An example of correlation function for the associated normal field has been given, for instance, by the Probabilistic Model Code [27], but at the present stage there is no agreement on this aspect of concrete strength distribution, as it will be pointed out next in this paragraph. Non-Gaussian and non-lognormal random fields are not covered by the procedure since they are rarely used for modeling the spatial distribution of concrete strength.

The assumption of homogeneity is again quite common in the context of reliability analysis of concrete structures [22, 23, 24, 26, 28, 27]. In certain cases this hypothesis may however turn out to be not appropriate, in particular if concretes with very different properties are found in a structure or in the case of vertical structural elements. In particular, if the practitioner identifies the presence of concrete batches with different properties in the structure, he should conduct different investigations for each of the various concrete materials identified, a provision that is already given by e.g. the ACI 214.4R standard. The variability in space of the random field properties may however also be due to the action of gravity induced pressures during the concreting and hardening phases, a case that is typical of vertical elements. It is recognized that this may cause a slight reduction of the average strength of concrete along the height. This aspect is typically ignored even in current codes for the assessment of concrete strength, and at best is taken into account by requiring the selection of a random sampling scheme. However, in the current stage of development, this phenomenon is not taken into consideration, and it will be the object of future investigations.

Finally, a discussion should be carried out on the assumption of a known correlation law. No information on this property of concrete strength are currently known with a satisfactory accuracy. Contradictory data and assumptions can be found in literature, and very few are based on experimental evidence. Some of the references seem to suggest little to no correlation at typical minimum inter-core distances (e.g. [22, 29]), whereas other works hypothesized the presence of more marked correlation functions. One of the very few experimental study on compressive strength correlation laws is due to Rackwitz and Müller [30]. On RC slabs they recorded a spatial correlation of concrete strength decaying to approximately zero after approximately 10 meters of relative distance between any two test locations, whereas on concrete roads they observed a persisting correlation even at hundreds of meters of distance. Vu and Stewart [22] conversely assumed in one of their papers a very rapidly decaying linear correlation law with zero correlation after a relative distance of $0.5m$, whereas in other works [23] the same authors modeled concrete compressive strength using a Gaussian correlation law with a correlation length of $2m$. The same assumption has been used by Firouzi and Rahai [26], whereas Tang et al. [29] modeled concrete strength by making use of an exponentially decaying law with scale of fluctuation between $0.4m$ and $4.0m$. Finally, the well-known Probabilistic Model Code models concrete strength through the use of a lognormal, homogeneous random field assuming a correlation law for the associated Gaussian field that never decays below 0.5. In figure 1 the different assumptions on the correlation laws of concrete strength that can be found in literature are depicted. The correlation of the lognormal field assumed by the Probabilistic Model Code for a standard deviation of $5.25MPa$ has been obtained by using the concepts stated in [31], to map the correlation of the associated normal field to that of the lognormal one.

By observing the figure, it is clear how at the present stage of knowl-

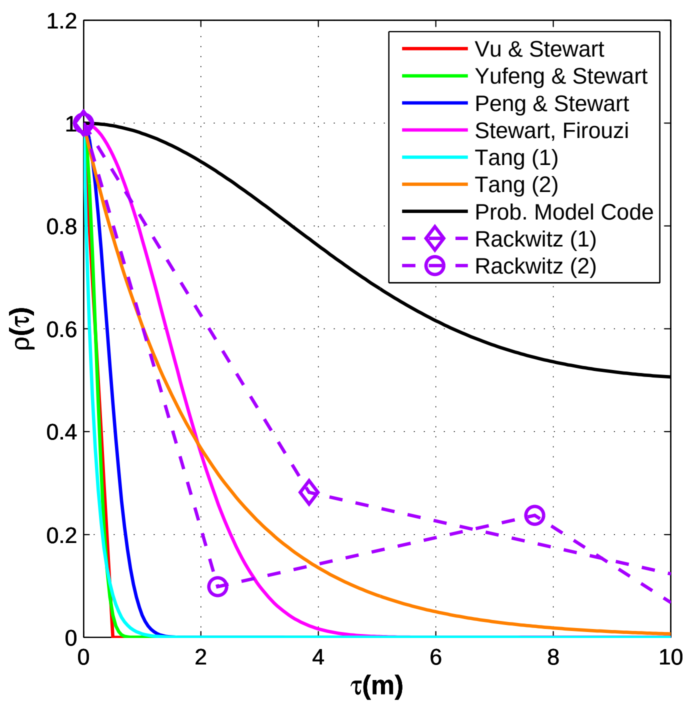

Figure 1: Comparison of several correlation functions associated to the spatial distribution of concrete compressive strength that can be found in literature. The dashed line represents data obtained from experimental campaigns.

edge no conclusions on the nature and intensity of the correlation of concrete strength can be made. Thus it seems arguable the a-priori assumption of negligible correlation. Within this setting, the proposed methodology generalizes the tolerance factor approach of the ACI 214.4R [7] making it useful to be applied to potentially any correlation function. Even though the knowledge of the correlation law is still required, this weakness is similar to that occurring with current approaches which implicitly assume uncorrelated samples (i.e. they assume a white-noise correlation function). The method thus offers more flexibility than current approaches with no further requirements.

3.3. Notation

Some notation needs to be introduced. It is assumed that n experimental values $\mathbf{x}_i$ with $i = 1 \ldots n$ are collected from an homogeneous Gaussian random field at locations r_i with $i = 1 \ldots n$. The mean of the field is denoted by μ whereas its standard deviation is indicated with σ. Both these two latter values are assumed to be unknown. The random column vector of observations is denoted by $\boldsymbol{X}$:

$$\boldsymbol{X} = (\mathbf{x}_1, \ldots, \mathbf{x}_n)^T \tag{12}$$

A 1xn row vector of weights W is defined as $W = (n^{-1}, \ldots, n^{-1})$. The sample mean is thus given by:

$$\bar{\mathbf{x}}_s = n^{-1} \sum x_i = W\boldsymbol{X} \tag{13}$$

The sample standard deviation can be expressed as:

$$\mathbf{s}_s = \sqrt{\frac{\sum(\mathbf{x}_i - \bar{\mathbf{x}}_s)^2}{n-1}} = \sqrt{\frac{\boldsymbol{X}^T(I_n - \mathbf{1}W)\boldsymbol{X}}{n-1}} \tag{14}$$

where I_n is the nxn identity matrix and $\mathbf{1} = (1, \ldots, 1)^T$ is a nx1 column vector of ones.

Since it is assumed that the random field is homogeneous, the correlation law, assumed to be known, is expressed as a function of the relative distance between any two points of the field:

$$\rho(r_i, r_j) = \rho(r_i - r_j) \tag{15}$$

The sample correlation matrix is thus given by:

$$C = \begin{bmatrix} \rho(r_1 - r_1) & \cdots & \rho(r_1 - r_n) \\ \vdots & \ddots & \vdots \\ \rho(r_n - r_1) & \cdots & \rho(r_n - r_n) \end{bmatrix} \tag{16}$$

and the sum of its entries is denoted by c^*:

$$c^* = \sum_{i,j} \rho(r_i - r_j) \tag{17}$$

Finally, the term ρ_m is used to represent the average of the out-of-diagonal terms of the C matrix:

$$\rho_m = \frac{\sum_{i \neq j} \rho(r_i - r_j)}{n(n-1)} = \frac{c^* - n}{n(n-1)} \tag{18}$$

3.4. Theoretical derivation

Equation (11) can be equivalently rewritten as:

$$Pr\left\{\frac{\bar{\mathbf{x}}_s - \mu}{\sigma} - \frac{k\mathbf{s}_s}{\sigma} \leq \frac{f_p - \mu}{\sigma}\right\} = 1 - \alpha \tag{19}$$

Exploiting the properties of the normal distribution, the preceding is equivalent to:

$$Pr\left\{\frac{\bar{\mathbf{x}}_s - \mu}{\sigma} - \frac{k\mathbf{s}_s}{\sigma} \leq z_p\right\} = 1 - \alpha \tag{20}$$

where it is recalled that z_p is the p^{th} percentile of the standard normal distribution. A normal random variable $\mathbf{z}$ is now defined as:

$$\mathbf{z} = \frac{\bar{\mathbf{x}}_s - \mu}{\sigma} \tag{21}$$

Equation (20) can then be rewritten as:

$$Pr\left\{\frac{\mathbf{z} + z_{1-p}}{\mathbf{s}_s/\sigma} \leq k\right\} = 1 - \alpha \tag{22}$$

It is well-known [32] that the random vector $\boldsymbol{X}$ can be expressed by means of another vector $\boldsymbol{Y}$ of independent standard normal random variables using a proper decomposition of the correlation matrix. If the principal square root matrix decomposition is chosen, then $\boldsymbol{X}$ is given by:

$$\boldsymbol{X} = \sigma C^{1/2}\boldsymbol{Y} + \mathbf{1}\mu \tag{23}$$

If Equation (23) is replaced in Equation (13) and in Equation (14) the followings are obtained:

$$\bar{\mathbf{x}}_s = \sigma W C^{1/2}\boldsymbol{Y} + W\mathbf{1}\mu = \sigma W C^{1/2}\boldsymbol{Y} + \mu \tag{24}$$

$$\mathbf{s}_s = \sigma\sqrt{\frac{\boldsymbol{Y}^T C^{1/2}(I_n - \mathbf{1}W)C^{1/2}\boldsymbol{Y}}{n-1}} = \sigma\sqrt{\frac{\boldsymbol{Y}^T B\boldsymbol{Y}}{n-1}} \tag{25}$$

In the latter equation the B matrix has been defined as:

$$B = C^{1/2}(I_n - \mathbf{1}W)C^{1/2} \tag{26}$$

The spectral decomposition of the B matrix is given by:

$$B = Q^T\Lambda Q \tag{27}$$

where Λ is the diagonal matrix of the eigenvalues λ_i of B and Q is the eigenvector matrix. It results that:

$$\mathbf{s}_s = \sigma\sqrt{\frac{(Q\boldsymbol{Y})^T \Lambda Q\boldsymbol{Y}}{n-1}} \tag{28}$$

It is useful to remark that by definition, B and $(I_n - \mathbf{1}W)$ are congruent, thus due to the Sylvester's theorem of inertia they share the same number of zero eigenvalues. Since $(I_n - \mathbf{1}W)$ is not a full rank matrix then at least one of the eigenvalues λ_i of B will be equal to zero, let's say $\lambda_n = 0$.

Furthermore, is easy to show (see Appendix A) that the random variables of the vector $Q\boldsymbol{Y}$ are independent and normally distributed with zero mean and variance equal to one. This observation allows to derive the known result that the sample variance is distributed as a linear combination of $n-1$ independent 1-dof Chi-Squared random variables $\boldsymbol{\chi}^2_{1,i}$:

$$\mathbf{s}_s^2 = \sigma^2 \frac{(Q\boldsymbol{Y})^T \Lambda Q\boldsymbol{Y}}{n-1} \sim \frac{\sigma^2}{n-1}\sum_{i=1}^{n-1} \lambda_i \boldsymbol{\chi}^2_{1,i} \tag{29}$$

Recalling Equation (24), the random variable $\mathbf{z}$ defined in Equation (21) is expressed as:

$$\mathbf{z} = WC^{1/2}\boldsymbol{Y} \tag{30}$$

Replacing Equations (28) and (30) in Equation (22), the following is obtained:

$$Pr\left\{\frac{WC^{1/2}\boldsymbol{Y} + z_{1-p}}{\sqrt{(Q\boldsymbol{Y})^T \Lambda Q\boldsymbol{Y}}} \le \frac{k}{\sqrt{n-1}}\right\} = 1-\alpha \tag{31}$$

Now an important result has been obtained. Equation (31) does not depend anymore on the unknown parameters of the field μ and σ, thus pivotal quantities have been derived. It is convenient to express this latter equation in a slightly more complex but equivalent form:

$$Pr\left\{\frac{\left(WC^{1/2}\boldsymbol{Y}+z_{1-p}\right)\sqrt{n^2/c^*}}{\sqrt{(Q\boldsymbol{Y})^T\Lambda Q\boldsymbol{Y}}}\leq k\sqrt{\frac{n^2}{(n-1)c^*}}\right\}=1-\alpha \tag{32}$$

Since the only unknown quantity in this expression is k, this can be obtained by solving the equation for it. To do so, a new random variable $\mathbf{u}$ is defined as:

$$\mathbf{u}=\frac{\left(WC^{1/2}\boldsymbol{Y}+z_{1-p}\right)\sqrt{n^2/c^*}}{\sqrt{(Q\boldsymbol{Y})^T\Lambda Q\boldsymbol{Y}}} \tag{33}$$

Its inverse cumulative distribution function evaluated at a generic value x is given by F_x^{-1}. Recalling Equation (18), the desired expression for k is thus finally given by:

$$k=k(n,p,\alpha,C)=F_{1-\alpha}^{-1}\sqrt{\frac{(n-1)[(n-1)\rho_m+1]}{n}} \tag{34}$$

Unfortunately a closed form solution for F_x^{-1} is, in general, not available, so that it must be numerically evaluated. Equation (34) also allows to intuitively understand the primary role that the average level of correlation ρ_m has on k. As the samples becomes more correlated (i.e. ρ_m increases), all other things being equal, k tends to grow, reaching in the limit $+\infty$ if all the samples are completely correlated (a situation in which there is simply not enough information to make any kind of estimation on the variance of the field). Nonetheless, it is important to remark that the effect of correlation on the estimates cannot be completely described just in terms of the synthetic parameter ρ_m, as F_x^{-1} depends on the whole decomposition $C^{1/2}$ of the correlation matrix.

It can further be shown that the presented theory is a generalization of the well-known Equation (2), which is obtained if $C = I_n$, as shown in Appendix B.

3.5. Approximate expression for the lower tolerance limit

Practical difficulties may arise evaluating k using the aforementioned expression. In particular the necessity of numerical procedures for the evaluation of the spectral decomposition of B and of the inverse CDF of $\mathbf{u}$ is certainly not convenient and may potentially limit the usefulness of the procedure for practical applications. It is thus desirable to obtain approximate but simple expressions for k to overcome these complex computations. Recalling Equation (29), the aim is to express the distribution of the sample variance as that of a single Chi-Squared random variable. This can be accomplished by making use of the Welch-Satterthwaite approximation [33, 34], that for this special case takes the following form:

$$\sum_{i=1}^{n-1} \lambda_i \boldsymbol{\chi}_{1,i}^2 \approx a \boldsymbol{\chi}_b^2 \tag{35}$$

where the scaling factor a and the degrees of freedom b of the single Chi-Squared distribution can be evaluated as:

$$a = \frac{\sum_{i=1}^{n-1} \lambda_i^2}{\sum_{i=1}^{n-1} \lambda_i} \quad \text{and} \quad b = \frac{\left(\sum_{i=1}^{n-1} \lambda_i\right)^2}{\sum_{i=1}^{n-1} \lambda_i^2} \tag{36}$$

Making use of the approximation (35) it is obtained that:

$$(Q\boldsymbol{Y})^T \Lambda Q \boldsymbol{Y} \approx a \boldsymbol{\chi}_b^2 \tag{37}$$

Using the previous result within the definition of $\mathbf{u}$ given by Equation (33) and noting that $WC^{1/2}\boldsymbol{Y}\sqrt{n^2/c^*} \sim N(0,1)$, the following is obtained:

$$\mathbf{u} \approx \frac{1}{\sqrt{ab}} \frac{N(0,1) + z_{1-p}\sqrt{n^2/c^*}}{\sqrt{\boldsymbol{\chi}_b^2/b}} \tag{38}$$

It is however necessary to note that the normal standard distribution at nominator is, in general, not independent from the Chi-Squared distribution at the denominator because if the samples are correlated their mean and variance are, in general, not independent. However numerical simulations have shown that typically the degree of dependency is very modest and can be reasonably neglected (see Appendix C). If this assumption is made, then by definition of non-central t distribution Equation (38) becomes:

$$\mathbf{u} \sim \frac{1}{\sqrt{\sum_{i=1}^{n-1} \lambda_i}} \boldsymbol{t}_b(z_{1-p}\sqrt{n^2/c^*}) \tag{39}$$

where $\boldsymbol{t}_x(y)$ is the non-central t distribution with x degrees of freedom and non-centrality parameter y. An approximate expression for k is finally given by:

$$k \approx t_{b,1-\alpha}^{-1}\left(z_{1-p}\sqrt{n^2/c^*}\right)\sqrt{\frac{(n-1)[(n-1)\rho_m + 1]}{n\sum_{i=1}^{n-1}\lambda_i}} \tag{40}$$

If the matrix $\bar{C}$ is defined as:

$$\bar{C} = \mathrm{tr}\{C(I_n - \mathbf{1}W)\} \tag{41}$$

then the explicit calculation of the spectral decomposition of B is not necessary and can be replaced with the more simple evaluation of the trace of $\bar{C}$ and of $\bar{C}^2$:

$$\sum_{i=1}^{n-1} \lambda_i = \mathrm{tr}(B) = \mathrm{tr}\{C^{1/2}(I_n - \mathbf{1}W)C^{1/2}\} = \mathrm{tr}\{C(I_n - \mathbf{1}W)\} = \mathrm{tr}(\bar{C}) \tag{42}$$

$$\sum_{i=1}^{n-1} \lambda_i^2 = \mathrm{tr}(B^2) = \mathrm{tr}\{C(I_n - \mathbf{1}W)C(I_n - \mathbf{1}W)\} = \mathrm{tr}(\bar{C}^2) \tag{43}$$

4. Examples of application of the proposed methodology

In this section two simple case-studies will be presented to investigate the different effects that a spatial correlation of samples may have on experimental investigations.

In the first case study the achieved confidence of estimation in presence of spatially correlated core strength values using the approaches of current standards is investigated and compared to the aforementioned methods. Quantifying the loss of confidence versus the target one is not a trivial task, because it essentially depends on the nature of each single problem (the spatial configuration of the cores, the specific distribution properties of concrete, etc.) and on the correlation properties of in-situ concrete, so that a single case study cannot cover all the possible situations. Despite these difficulties, the analysis of a simplified scenario still results to be helpful to identify the potential risks of neglecting the correlation of core test values.

In the second case study it is shown how the presence of a spatial correlation may result in situations in which there is an hard limit on the accuracy of estimates regardless of the number of samples extracted. In such cases it results that there is no benefit whatsoever in further increasing the number of cores above a certain limit.

4.1. Case study 1

The first case study consists in the evaluation of a given percentile of concrete strength for a rectangular RC slab with a plan dimensions of 8x24m.

As a common practice in reliability analysis of RC structures, in-situ concrete compressive strength is assumed to be a realization of a homogeneous Gaussian random field with squared exponential correlation law of the type $\rho(x) = e^{-(x/d)^2}$, where d is a parameter that has been assumed to vary between $0.5m$ and $7.5m$. The average concrete strength is set to be equal to $35MPa$ and the coefficient of variation is 0.15. It is also assumed that 12 cores are extracted in a grid layout with row and column spacing of $4m$, as depicted in Figure 2.

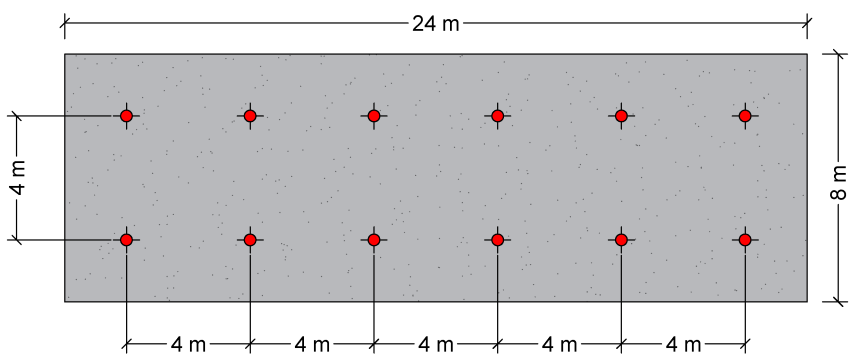

Figure 2: Case study 1 - Position of the concrete cores.

Correlated samples have been obtained using a very simple covariance decomposition method, and for each sample set an estimate has been obtained applying the criteria of the different codes and the approaches proposed in the previous sections.

Repeating the simulation 400000 times, a dataset of estimates has been obtained and applying a Monte-Carlo approach the confidence (as probability of underestimation of the actual percentile) has then been evaluated. The number of simulations has been estimated using the binomial proportion confidence intervals [35], i.e. by requiring the actual estimated percentile to be within 0.1% from the desired one with 95% confidence.

For the ACI 214.4R the influence of the strength correction factors has been neglected by assuming $s_a = 0$ and for the application of the alternate method it has been assumed that concrete samples were coming from a single batch. The chosen target confidence has been set to 90%.

In Figure 3 is depicted the confidence achieved by applying the ACI 214.4R code requirements in comparison with that of proposed theory and the approximate expression of Eq.(40).

Analyzing the results it is clear how the alternate approach of ACI 214.4R consistently yields low confidence results regardless of the actual level of spatial correlation. This was expected as this method has been developed to produce less conservative results, but the drop in the theoretical achieved confidence appears to be so marked that it is Authors' opinion that its appropriateness should at least be further investigated.

The classic tolerance factor method instead produces exact or very good results when no correlation or a low level of spatial correlation among samples is present. Nevertheless, if the spatial correlation is consistent, its confidence can drop down to values significantly lower than the desired one.

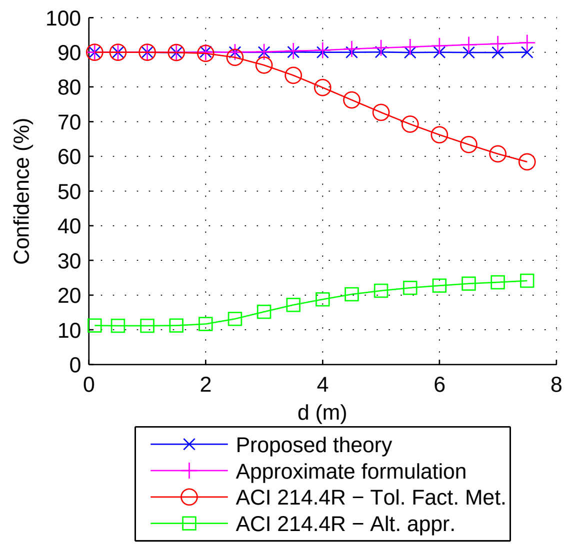

Figure 3: Case study 1 - Actual confidence achieved in the estimation of the 10% percentile of concrete strength as a function of the correlation length d using the ACI 214.4R approaches and the proposed exact and approximate theoretical formulations.

The proposed theory as expected always produces a confidence equal to the target one regardless of the level of spatial correlation, if the correlation law is known. The approximate formulation, that allows avoiding complex numerical computations, still yields very good results that are on the safe side as the achieved confidence values are slightly higher than the target one.

In Figure 4 the confidence in the estimation of the 5% percentile of concrete strength achieved by the approach of EN13791 is depicted in comparison to the proposed theoretical formulations.

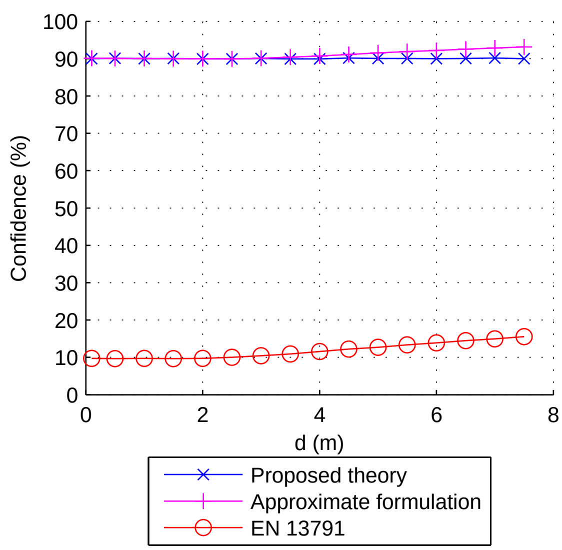

Figure 4: Case study 1 - Actual confidence in the estimation of the 5% percentile as a function of the correlation length d achieved by the approach of EN 13791 and the proposed exact and approximate theoretical formulations.

The theoretical formulation produces again a confidence exactly equal to the target one, while the approximate approach produces results that are very close to the desired confidence. However, the confidence in the

EN13791 estimates constantly remains below the 20% mark regardless of the investigated correlation length d. It is Authors' opinion that the approach of the EN standard should be revised since such low confidences may produce concrete strength evaluations (and ultimately structural assessments) that are not consistent with the acceptable probability of failure that is behind the calibration of currently used Limit State approach.

Different conclusions are drawn if the approach of Eurocode 8 is used. This code deals with the evaluation of the average strength and, as it has been stated, the actual estimation depends on the knowledge level. Since this latter depends on a variety of factors not all related to the experimental testing of material properties, the outcomes corresponding to the three different knowledge levels have been investigated.

From Figure 5 it is clear that if the best knowledge level (KL3) is reached, then the confidence actually drops because the corresponding confidence factor is 1.00, which assumes that there are no uncertainties on the knowledge of the structure, a state that can never be achieved. If the knowledge level is lower, than the estimation of the average concrete strength is highly conservative because the sample mean is divided by a factor 1.20 or 1.35 respectively for $KL2$ and $KL3$. This results in estimates that reach levels of confidence very close to 100%. Nevertheless, this result is still misleading since these very high confidence levels, that correspond to very low strength values, have the duty of taking into account also the uncertainties not related to the knowledge of structural material. It is thus clear that for the case of EC8 and KL2 or KL3 is very hard to make any kind of judgment on the appropriateness of the empirical formulation. For what concerns the

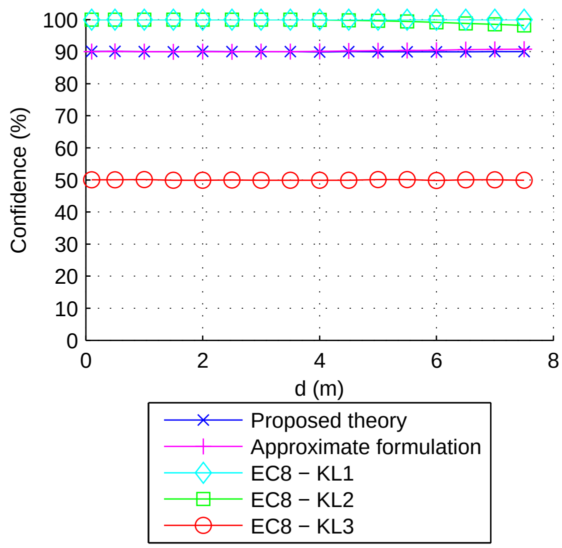

Figure 5: Case study 1 - Actual confidence in the estimation of the average concrete strength as a function of the correlation length d achieved by the approach of EC8 and the proposed exact and approximate theoretical formulations.

influence of spatial correlation, this latter is practically absent, and the level of confidence depends almost only on the chosen confidence factors.

4.2. Case study 2

In this second case study the attention is focused on the number of cores to be drilled. In particular it is desired to find whether it exists a maximum number of cores above which it is pointless to go, and if so how this value is affected by spatial correlation. The analysis will be carried out on an ideal monodimensional structure of $50m$ of length, which may ideally represent, for example, an old multi-span RC slab bridge.

It is supposed that n cores are collected along the longitudinal direction of the bridge at equal intervals determined in order to maximize the inter-core distance. This means that if 6 samples are collected they are spaced $10m$ apart one from each other, whereas if 11 cores are extracted their mutual distance is $5m$.

The number of samples may vary between 5 and 25, i.e. their spacing ranges from $8.3m$ to roughly $2.0m$. It is also assumed that the in-situ concrete strength follows a Gaussian correlation law e^{-x^2/d^2} with the parameter d starting from a very low value of $0.05m$ and then ranging between $0.5m$ and $8.0m$, so that the influence of the degree of correlation on the problem can be investigated.

The mechanical properties of concrete strength are the same as those of case study 1, i.e. an average compressive strength of $35MPa$ and a coefficient of variation of 0.15.

The analysis consist in simulating, for each considered number n of cores, 2 millions of different correlated samples draws.

For the i^{th} sampling with correlation length d, the average strength of samples is denoted by $\bar{x}_{s,i,d}$ whereas their standard deviation is given by $s_{s,i,d}$. The associated estimation $f_{10,est,i,d}$ of the 10% percentile f_{10} is then carried out using the aforementioned method:

$$f_{10,est,i,d} = \bar{x}_{s,i,d} - k s_{s,i,d} \tag{44}$$

with k evaluated using equation (34) and fixing a confidence of 90%. Using a Monte-Carlo approach the average overestimation $\bar{f}_{10,+}$ of f_{10} is estimated by:

$$\bar{f}_{10,+} = \frac{\sum_{i=1}^{2\cdot 10^6} f_{10,est,i,d}[f_{10,est,i,d} > f_{10}]}{\sum_{i=1}^{2\cdot 10^6} 1[f_{10,est,i,d} > f_{10}]} - f_{10} \tag{45}$$

whereas the dispersion (i.e. the standard deviation) of the overestimating values is given by:

$$s_{10,+} = \sqrt{\frac{\sum_{i=1}^{2\cdot 10^6} (f_{10,est,i,d} - f_{10} - \bar{f}_{10,+})^2 [f_{10,est,i,d} > f_{10}]}{\sum_{i=1}^{2\cdot 10^6} 1[f_{10,est,i,d} > f_{10}] - 1}} \tag{46}$$

The average percentile underestimation $f_{10,-}$ and the standard deviation of the estimations lower than f_{10} are similarly defined as:

$$\bar{f}_{10,-} = \frac{\sum_{i=1}^{2\cdot 10^6} f_{10,est,i,d}[f_{10,est,i,d} \leq f_{10}]}{\sum_{i=1}^{2\cdot 10^6} 1[f_{10,est,i,d} \leq f_{10}]} - f_{10} \tag{47}$$

$$s_{10,-} = \sqrt{\frac{\sum_{i=1}^{2\cdot 10^6} (f_{10,est,i,d} - f_{10} - \bar{f}_{10,-})^2 [f_{10,est,i,d} \leq f_{10}]}{\sum_{i=1}^{2\cdot 10^6} 1[f_{10,est,i,d} \leq f_{10}] - 1}} \tag{48}$$

In the four aforementioned equations the square brackets represent the Iverson notation.

In Figures 6 and 7 are respectively depicted the average overestimation error $\bar{f}_{10,+}$ and the standard deviation $s_{10,+}$ of the overestimating values.

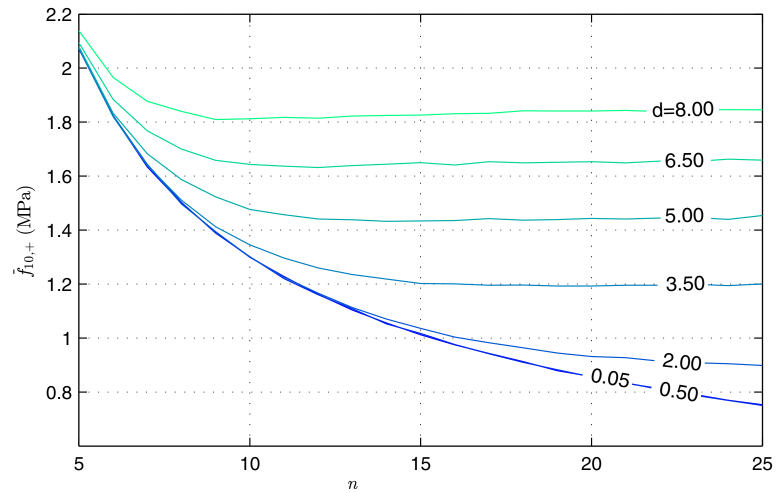

Figure 6: Case study 2 - Average overestimation $\bar{f}_{10,+}$ of the 10^{th} percentile as a function of the number of samples n and the correlation length d.

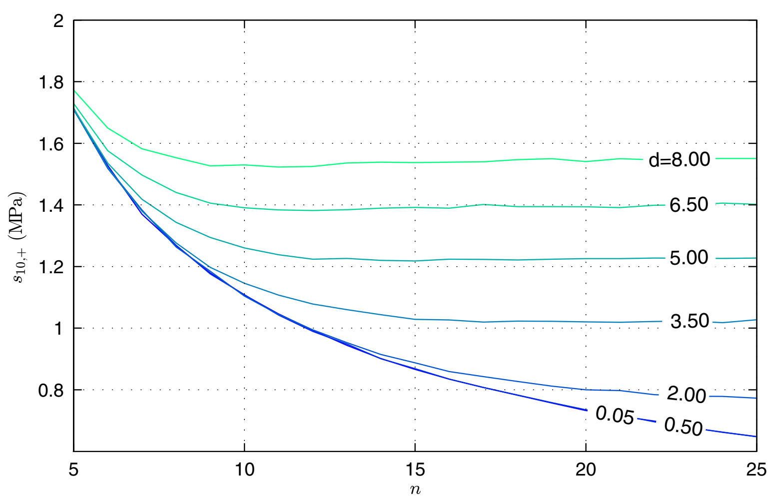

Figure 7: Case study 2 - Standard deviation $s_{10,+}$ of the estimations above the actual 10^{th} percentile as a function of the number of samples n and the correlation length d.

From these figures it is clear that, as the sample spacing decreases (n increases) it becomes less convenient to increase the number of samples to reduce the amount of overestimation. If even a modest spatial correlation is present, it can be clearly seen from Figure 6 that there exists a number of cores (as a function of d) above which the average overestimation error (which is realized in 10% of cases) does not decrease anymore. At the same time even the dispersion of the overestimating values around their mean does not improve.

Similar conclusions can be drawn if the attention is focused towards the underestimating values. In this case it is desirable that the underestimation is as close as possible to zero, so not to risk ending up using too conservative values. As shown in Figures 8 and 9, the results suggest that as the degree of correlation of samples increases, it is less and less useful to increase the number of cores with a decreasing inter-distance, since below a certain spacing there is practically no advantage both in terms of average underestimation and dispersion of the underestimating values.

As a result this second case study induces to believe that if even a modest degree of correlation is recorded then there is an ideal number of cores (and a linked inter-core distance) above which no useful improvements of the accuracy of estimations can be further achieved.

5. Conclusions

The analysis of some of the principal international approaches currently in use for the statistical interpretation of in-situ concrete core strength highlighted the lack of a universally accepted method for the evaluation of this

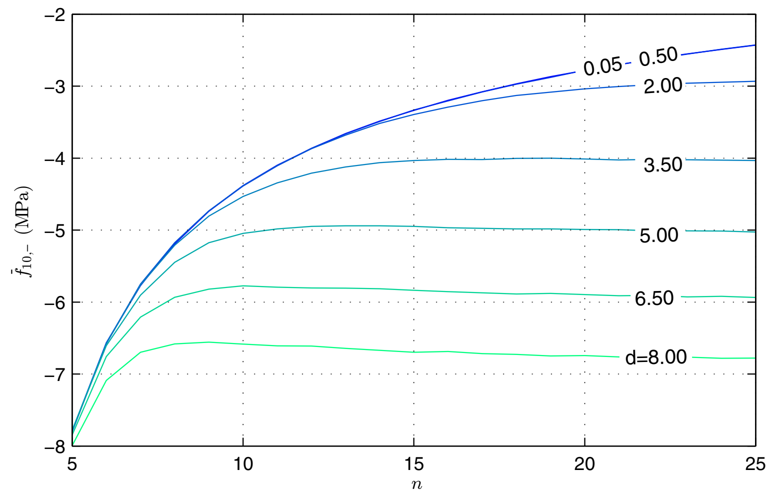

Figure 8: Case study 2 - Average underestimation $\bar{f}_{10,-}$ of the 10^{th} percentile as a function of the number of samples n and the correlation length d.

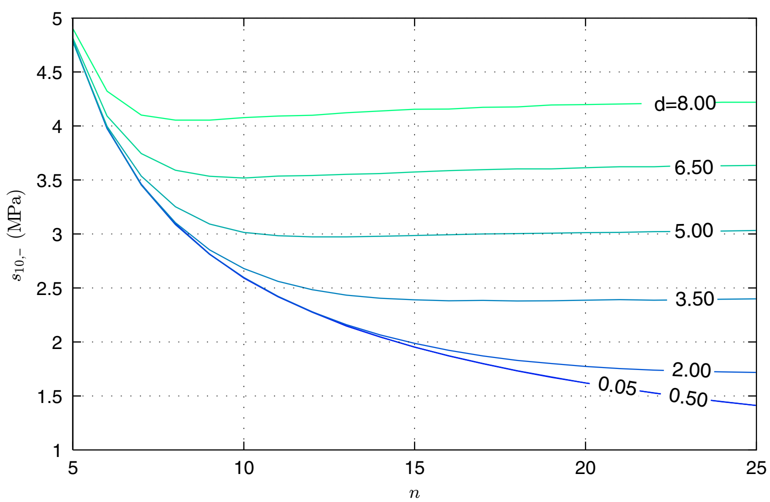

Figure 9: Case study 2 - Standard deviation $s_{10,-}$ of the estimations below the actual 10^{th} percentile as a function of the number of samples n and the correlation length d.

property. Most of these are based on empirical criteria generally not supported by a solid theoretical background, and all of them rely on assumptions on concrete strength distribution which, even though commonly accepted, are not supported or universally accepted by current scientific literature. In particular, the assumption of independence of in-situ concrete strength is not supported with sufficient evidence by the existing literature. Many authors and even well-known references such as the Probabilistic Model Code are currently modeling the spatial strength distribution of this material using widely different assumptions on the levels of spatial correlation.

Given this state of the art, it has been considered useful to set up a sound statistical framework to perform evaluations of concrete strength valid regardless of the assumed level of correlation. The developed approach is a generalization of the well-known tolerance factors method used by the ACI 214.4R to make it suitable to be applied to any level of correlation. At this stage the theory relies on the hypothesis of concrete strength distributed as an homogeneous Gaussian random field with known correlation function, which are usually accepted in the context of structural reliability. As the theoretical formulation is quite complex in the case of a generic correlation function, simpler approximate expressions have been derived.

The approach allows to properly take into consideration the reduction in the level of information on the underlying field that occurs when the strength values are spatially correlated. However further experimental studies are required to carefully evaluate the actual spatial stochastic properties of concrete strength distribution before a practical application of these formulas can be made. Nonetheless, this evaluation is highly desirable regardless of the actual approach to the interpretation of core test data that is being used, as it cannot be a-priori assumed that the correlation is negligible as it is currently done.

In the second part of the work some case studies have been presented to highlight some of the potential implications that might occur in the case the levels of correlation turn out to be not-negligible. A first case study suggested that if a significant spatial correlation is present then the use of current approaches may result in very high probability of overestimation of the desired percentile, thus exposing the consequent structural evaluations to reliability issues. The proposed theoretical formulations instead allowed to obtain evaluations matching or very close to the desired confidence levels. The second case study investigated the influence of the number of cores (and the consequent inter-core distance) on the accuracy of the evaluations on a mono-dimensional structure if a spatial correlation of in-situ concrete strength is present. The results seem suggesting that in presence of even a modest correlation there exist a minimum core inter-distance, depending on the degree of correlation, below which the accuracy cannot improve anymore. This observation may have practical consequences on the estimation of the desired number of cores to extract in real-world scenarios.

Appendices

Appendix A

From the definition of covariance, the covariance matrix $R_{Q\boldsymbol{Y}}$ of the random vector $Q\boldsymbol{Y}$ is given by:

$$R_{Q\boldsymbol{Y}} = E\left[\left(Q\boldsymbol{Y} - E[Q\boldsymbol{Y}]\right)\left(Q\boldsymbol{Y} - E[Q\boldsymbol{Y}]\right)^T\right] \tag{49}$$

Since the random variables in $\boldsymbol{Y}$ are independent standard normals their mean $E[\boldsymbol{Y}]$ is the nx1 null vector, and their covariance matrix is the identity matrix I_n. It results that:

$$R_{Q\boldsymbol{Y}} = E\left[Q\boldsymbol{Y}\boldsymbol{Y}^T Q^T\right] = QE[\boldsymbol{Y}\boldsymbol{Y}^T]Q^T = QI_nQ^T = QQ^T = I_n \tag{50}$$

In the last equation the orthogonality of the eigenvector matrix Q has been exploited. As a result it is thus clearly shown that the random variables of the $Q\boldsymbol{Y}$ vector are independent standard normal random variables.

Appendix B

If the samples are uncorrelated then $C = I_n$, and consequently it results that:

$$WC^{1/2}\boldsymbol{Y}\sqrt{n^2/c^*} = W\boldsymbol{Y}\sqrt{n} \sim N(0,1) \tag{51}$$

Furthermore $B = I_n - \mathbf{1}W$ is a circulant matrix with $n-1$ eigenvalues equal to 1 and one eigenvalue equal to zero. Thus:

$$\sqrt{(Q\boldsymbol{Y})^T \Lambda Q\boldsymbol{Y}} \sim \sqrt{\sum\nolimits_{i=1}^{n-1} \lambda_i \boldsymbol{\chi}^2_{1,i}} = \sqrt{\boldsymbol{\chi}^2_{n-1}} \tag{52}$$

Replacing Equations (51) and (52) in Equation (32) it results:

$$Pr\left\{\frac{N(0,1)+z_{1-p}\sqrt{n}}{\sqrt{\frac{\boldsymbol{\chi}^2_{n-1}}{n-1}}}\leq k\sqrt{n}\right\}=1-\alpha \tag{53}$$

Since in the case of independent samples the sample mean and standard deviation are independent, the distributions at nominator and denominator are independent one to each other and thus the first term of the inequality of Equation (53) follows a non-central t-distribution with $n-1$ degrees of freedom and non-centrality parameter $z_{1-p}\sqrt{n}$.

$$Pr\left\{t_{n-1}(z_{1-p}\sqrt{n})\leq k\sqrt{n}\right\}=1-\alpha \tag{54}$$

Using the properties of the inverse cumulative distribution function for the non-central t-distribution the probability operator can be dropped and it results that:

$$k=t^{-1}_{n-1,1-\alpha}(z_{1-p}\sqrt{n})/\sqrt{n} \tag{55}$$

This latter expression is exactly the same of Equation (2).

Appendix C

By making use of Equation (29) and noting that $WC^{1/2}\boldsymbol{Y}\sqrt{n^2/c^*}\sim N(0,1)$, Equation (33) can be rewritten in the following form:

$$\mathbf{u}=\frac{\left(WC^{1/2}\boldsymbol{Y}+z_{1-p}\right)\sqrt{n^2/c^*}}{\sqrt{\sum_{i=1}^{n-1}\lambda_i\boldsymbol{\chi}^2_{1,i}}} \tag{56}$$

The previous form is analogous to the one of Equation (33), but it should be noted that the linear combination of $\boldsymbol{\chi}^2$ distributions at denominator is, in general, not independent from the normal distribution at nominator, as they are function of the sample mean and variance which, in the case of correlated samples, are typically not independent. The degree of dependency however appears to be very weak, and this can qualitatively be observed for instance by plotting and comparing the CDFs associated with the random variable of Equation (33) and with that of (56) having assumed in this latter the independence between distributions at nominator and denominator.

As the distribution of $\mathbf{u}$ depends on the number of samples, on their relative position, on the required percentile and on the correlation function several analyses have been carried out. However, as examples, only two of these will be presented here, the first one related to a scenario with 3 samples and a required percentile $p = 0.5$ and the second one regarding a case with 15 samples and $p = 0.1$. In figure 10 the empirical CDFs for the two aforementioned cases are plotted. They have been obtained by using 10 millions samples, a value estimated using the same criteria stated in section 4.1 so to that the empirical percentile is approximately within 0.03% from the actual one with 95% confidence.

From the charts it can be seen how the assumption of the independence between the sample mean and variance has a negligible effect on the distribution of the random variable $\mathbf{u}$. Consequently this hypothesis has been judged acceptable in the setting of the approximate method presented in section 3.5.

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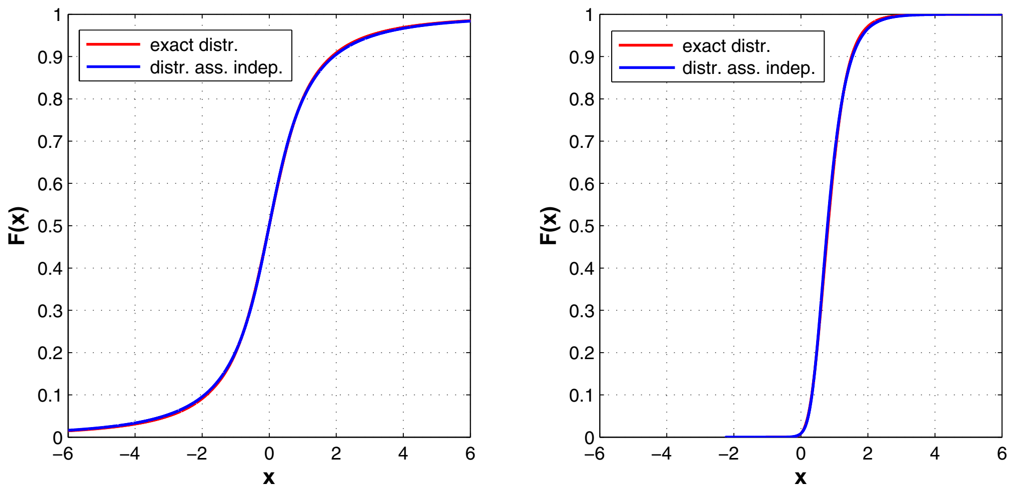

(a) $n = 3$, $p = 0.5$

(b) $n = 15$, $p = 0.1$

Figure 10: Comparisons between the actual CDF of $\mathbf{u}$ and the one obtained assuming the independence between the sample mean and variance for the two analyzed examples.

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